

thm_2Ecardinal_2ECARD_EQ_BIJECTION
(TMbFG-
WyQtA8LEYvxMFNuDzfkNu3K3o5QbKy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epred_set_2ECARD A_27a \in (ty_2Enum_2Enum^{(2^{A-27a})}) \quad (5)$$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \quad (7)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A-27a})^{((ty_2Epair_2Eprod A_27a 2)^{A-27b})}) \quad (8)$$

Definition 16 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A-27a}).(ap (c_2$

Definition 17 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 18 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E_21 2)$

Definition 19 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 20 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A-27b})^{A-27$

Definition 21 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A-27a}) A$

Definition 22 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap ($

Definition 23 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmic_2E_3C_3D V0m) V0m))) \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p(ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p(ap V0P V2x)))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\forall V2x \in A.27a.(p(ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A.27a.((p(ap V0P V3x)) \wedge (p V1Q)))))) \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \wedge (\forall V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a.((p V0P) \wedge (p(ap V1Q V3x)))))) \quad (28)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).(((p V0P) \vee (\exists V2x \in A.27a.(p(ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A.27a.((p V0P) \vee (p(ap V1Q V3x)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ 2.((\exists V2x \in A.27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q)))) \Leftrightarrow ((\exists V3x \in \\ A.27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A.27a}).((\exists V2x \in A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ 2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\ (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee \\ (p \ V0A)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\ p \ V0A)) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))))) \Rightarrow \\ ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\ \forall V0P \in ((2^{A.27b})^{A.27a}).((\forall V1x \in A.27a.(\exists V2y \in \\ A.27b.(p \ (ap \ (ap \ V0P \ V1x) \ V2y)))) \Leftrightarrow (\exists V3f \in (A.27b^{A.27a}).(\\ \forall V4x \in A.27a.(p \ (ap \ (ap \ V0P \ V4x) \ (ap \ V3f \ V4x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27b}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_27a)\ V0s)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1t)) \wedge (p \\
& \quad (ap\ (ap\ c_2Earithmic_2E_3C_3D\ (ap\ (c_2Epred_set_2ECARD\ A_27a) \\
& \quad \quad V0s))\ (ap\ (c_2Epred_set_2ECARD\ A_27b)\ V1t)))))) \Rightarrow (\exists V2f \in \\
& \quad (A_27b^{A_27a}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27b)\ (ap\ (\\
& \quad ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V2f)\ V0s))\ V1t)) \wedge (\forall V3x \in \\
& \quad A_27a.(\forall V4y \in A_27a.(((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x) \\
& \quad V0s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4y)\ V0s)) \wedge ((ap\ V2f\ V3x) = \\
& \quad \quad (ap\ V2f\ V4y)))))) \Rightarrow (V3x = V4y))))))))) \\
& \hspace{15em} (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27b}).(\forall V2f \in \\
& \quad (A_27b^{A_27a}).(((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \wedge \\
& \quad ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1t)) \wedge (((ap\ (c_2Epred_set_2ECARD \\
& \quad A_27a)\ V0s) = (ap\ (c_2Epred_set_2ECARD\ A_27b)\ V1t)) \wedge (p\ (ap\ (ap \\
& \quad (c_2Epred_set_2ESUBSET\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V0s))\ V1t)))))) \Rightarrow ((\forall V3y \in A_27b.((p\ (ap\ (\\
& \quad ap\ (c_2Ebool_2EIN\ A_27b)\ V3y)\ V1t)) \Rightarrow (\exists V4x \in A_27a.((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A_27a)\ V4x)\ V0s)) \wedge ((ap\ V2f\ V4x) = V3y)))))) \Leftrightarrow (\\
& \quad \forall V5x \in A_27a.(\forall V6y \in A_27a.(((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V5x)\ V0s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V6y)\ V0s)) \wedge \\
& \quad ((ap\ V2f\ V5x) = (ap\ V2f\ V6y)))))) \Rightarrow (V5x = V6y))))))))) \\
& \hspace{15em} (40)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Ecombin_2EI \\
\quad A_27a)\ V0x) = V0x)) \hspace{15em} (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))))) \\
& \hspace{15em} (42)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{15em} (43)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{15em} (44)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27b}).(((p\ (ap\ (c_2Epred_set_2EFINITE \\ & A_27a)\ V0s)) \wedge ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1t)) \wedge (\\ & ap\ (c_2Epred_set_2ECARD\ A_27a)\ V0s) = (ap\ (c_2Epred_set_2ECARD \\ & A_27b)\ V1t)))) \Rightarrow (\exists V2f \in (A_27b^{A_27a}).((\forall V3x \in A_27a. \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ (ap\ V2f\ V3x))\ V1t)))) \wedge ((\forall V4y \in A_27b.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27b)\ V4y)\ V1t)) \Rightarrow (\exists V5x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V5x)\ V0s)) \wedge ((ap\ V2f\ V5x) = V4y)))))) \wedge (\forall V6x \in A_27a. \\ & (\forall V7y \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V6x)\ V0s)) \wedge \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V7y)\ V0s)) \wedge ((ap\ V2f\ V6x) = (ap\ V2f \\ & V7y)))))) \Rightarrow (V6x = V7y)))))))))) \end{aligned}$$