

thm_2Ecardinal_2ECARD_LDISTRIB
(TMLfcu4kmaoWTUyL6vm6rJNHCttHWF9CxZZ)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))$

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (\text{ap } (\text{c_2Emin_2E_40 } A_27a$

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a$

Definition 10 We define `c_2Ebool_2E_3F_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Ebool_2E_2F_5C } A_27a$

Definition 11 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x))$

Definition 12 We define `c_2Epred_set_2ESURJ` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 13 We define `c_2Epred_set_2EINJ` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 14 We define `c_2Epred_set_2EBIJ` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a})$

Definition 15 We define `c_2Ecardinal_2Ecardeq` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s1 \in (2^{A_27a}). \lambda V1s2 \in (2^{A_27a})$

Definition 16 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_21 } 2))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (1)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (2)$$

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (3)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (4)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (x, y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (5)$$

Definition 19 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ s\ t)$

Definition 22 We define $c_2Ecardinal_2E_2B_c$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27b}).(ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ s\ t)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (6)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (7)$$

Definition 23 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).$

Let $c_2Esum_2EOUTR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTR \\ A_27a\ A_27b \in (A_27b^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (8)$$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in \\ A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Emin_2E_40\ A_27a)\ (\lambda V1y \in A_27a. (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x)\ V1y))) = V0x)) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V1x \in A_27a. (ap\ V0P\ V1x)))) \Leftrightarrow ((\exists V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (\forall V3x \in A_27a. (\forall V4y \in A_27a. (((p\ (ap\ V0P\ V3x)) \wedge (p\ (ap\ V0P\ V4y))) \Rightarrow (V3x = V4y))))))) \quad (23)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in A_27a. (\exists V1x \in A_27a. (V1x = V0a))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a)))))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ A_27a\ A_27b)\ V0s)\ V1t))) \Leftrightarrow (\exists V2f \in (A_27b^{A_27a}). ((\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V0s)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ V2f\ V3x))\ V1t)))))) \wedge (\forall V4y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V4y)\ V1t)) \Rightarrow (p\ (ap\ (c_2Ebool_2E_3F_21\ A_27a)\ (\lambda V5x \in A_27a. (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5x)\ V0s))\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27b)\ (ap\ V2f\ V5x))\ V4y)))))))))) \quad (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). ((\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Esum_2Esum\ A_27a\ A_27b))\ (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V2x))\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ V0s)\ V1t))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)))) \wedge \\ & (\forall V3y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Esum_2Esum\ A_27a\ A_27b))\ (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V3y))\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c\ A_27a\ A_27b)\ V0s)\ V1t))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V3y)\ V1t)))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). (\forall V2x \in A_27a. (\forall V3y \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V3y)\ V1t)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}). (\exists V1fn \in \\ & \quad (A_27c^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). (\forall V2x \in A_27a. (\\ & \quad \forall V3y \in A_27b. ((ap\ V1fn\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ \\ & \quad V2x)\ V3y)) = (ap\ (ap\ V0f\ V2x)\ V3y)))))) \\ & \hspace{15em} (33) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((\exists V1p \in \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p_1 \in \\ & \quad A_27a. (\exists V3p_2 \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ A_27b)\ V2p_1)\ V3p_2)))))) \\ & \hspace{15em} (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}). ((\forall V1p \in \\ & \quad (ty_2Epair_2Eprod\ A_27a\ A_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ & \quad A_27a. (\forall V3p_2 \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ A_27b)\ V2p_1)\ V3p_2)))))) \\ & \hspace{15em} (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap\ (c_2Esum_2EINL\ A_27a \\ & \quad A_27b)\ V0x) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V1y))) \Leftrightarrow (V0x = V1y))) \\ & \hspace{15em} (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27b. (\forall V1y \in A_27b. (((ap\ (c_2Esum_2EINR\ A_27a \\ & \quad A_27b)\ V0x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V1y))) \Leftrightarrow (V0x = V1y))) \\ & \hspace{15em} (37) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0y \in A_27a. (\forall V1x \in A_27a. (((ap\ (c_2Esum_2EINL \\ & \quad A_27a\ A_27b)\ V1x) = (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V0y))) \Leftrightarrow (V1x = \\ & \quad V0y))) \wedge (\forall V2y \in A_27b. (\forall V3x \in A_27b. (((ap\ (c_2Esum_2EINR \\ & \quad A_27a\ A_27b)\ V3x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V2y))) \Leftrightarrow (V3x = \\ & \quad V2y)))) \\ & \hspace{15em} (38) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}).((\forall V1s \in \\
& \quad (ty_2Esum_2Esum\ A_27a\ A_27b).(p\ (ap\ V0P\ V1s))) \Leftrightarrow ((\forall V2x \in \\
& \quad A_27a.(p\ (ap\ V0P\ (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V2x)))) \wedge (\forall V3y \in \\
& \quad A_27b.(p\ (ap\ V0P\ (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V3y))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}).((\exists V1s \in \\
& \quad (ty_2Esum_2Esum\ A_27a\ A_27b).(p\ (ap\ V0P\ V1s))) \Leftrightarrow ((\exists V2x \in \\
& \quad A_27a.(p\ (ap\ V0P\ (ap\ (c_2Esum_2EINL\ A_27a\ A_27b)\ V2x)))) \vee (\exists V3y \in \\
& \quad A_27b.(p\ (ap\ V0P\ (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V3y))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a.(\forall V1y \in A_27b.(\neg((ap\ (c_2Esum_2EINL \\
& \quad A_27a\ A_27b)\ V0x) = (ap\ (c_2Esum_2EINR\ A_27a\ A_27b)\ V1y))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27b.((ap\ (c_2Esum_2EOUTR\ A_27a\ A_27b)\ (ap\ (c_2Esum_2EINR \\
& \quad A_27a\ A_27b)\ V0x)) = V0x))
\end{aligned} \tag{42}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27b}). \\
& \quad (\forall V2u \in (2^{A_27c}).(p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ (ty_2Epair_2Eprod \\
& \quad A_27a\ (ty_2Esum_2Esum\ A_27b\ A_27c))\ (ty_2Esum_2Esum\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ (ty_2Epair_2Eprod\ A_27a\ A_27c)))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad A_27a\ (ty_2Esum_2Esum\ A_27b\ A_27c))\ V0s)\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c \\
& \quad A_27b\ A_27c)\ V1t)\ V2u)))\ (ap\ (ap\ (c_2Ecardinal_2E_2B_c\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)\ (ty_2Epair_2Eprod\ A_27a\ A_27c))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad A_27a\ A_27b)\ V0s)\ V1t))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27c) \\
& \quad V0s)\ V2u))))))
\end{aligned}$$