thm_2Ecardinal_2ECARD__LT__LE (TMX9VLpB1hTA2pV6mnHo3bnEq7JXqv95xdf)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.\text{c\_2Ebool\_2ET})$.

Definition 4 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Epred__set_2ESURJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 11 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Definition 12 *We define* c_2Epred__set_2EBIJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Definition 13 *We define* c_2Ecardinal_2Ecardeq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A\_}$

Definition 14 *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^A$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2E}$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{3}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).((\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1t)\ V0s))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V0s)\ V1t)) \wedge (\neg (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1t)\ V0s))))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1t)\ V0s))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardeq}\ A\_27a\ A\_27b)\ V0s)\ V1t))))) \tag{6}$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{7}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg (p\ V0A)) \Rightarrow False))) \tag{8}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \tag{9}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((\neg (p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg (p\ V1B)) \Rightarrow False))))) \tag{10}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{11}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))) \tag{12}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{13}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{14}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1t)\ V0s)))\Leftrightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V0s)\ V1t))\wedge(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardeq}\ A\_27a\ A\_27b)\ V0s)\ V1t))))))))$$