

thm_2Ecardinal_2ECOUNTABLE (TMax2DtxsLi9Z8vABajyVx2F3j4VCUuSuEH)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda V1f V0x)))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\lambda V1P \in (2^{A_27a}).(\lambda V1P (c_2Emin_2E_3D (2^{A_27a}))))))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\lambda V1P \in (2^{A_27a}).(\lambda V1P (ap (c_2Emin_2E_40 A) P))))$

Definition 10 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27b})$

Definition 11 We define $c_2Ecardinal_2Ecardgeq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27b})$

Definition 12 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_3F)$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 13 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(\lambda V1P \in (2^{A_27a}).(\lambda V1P (c_2Ebool_2E_3F)))$

Assume the following.

$$True \tag{2}$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_{.27a}.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Theorem 1

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0t \in (2^{A_{.27a}}).((p\ (ap\ (c_{.2}Epred_set_{.2}Ecountable\ A_{.27a})\ V0t)) \Leftrightarrow (p\ (ap\ (ap\ (c_{.2}Ecardinal_{.2}Ecardgeq\ ty_{.2}Enum_{.2}Enum\ A_{.27a})\ (c_{.2}Epred_set_{.2}EUNIV\ ty_{.2}Enum_{.2}Enum))\ V0t))))$$