

thm_2Ecardinal_2EFINITE_IMAGE_INJ (TMMPKy6iV8VYi3t93yL5NqLy5ouSduEpev3)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \tag{3}$$

Definition 9 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in$

Definition 11 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2E$

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 13 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21) 2)$

Definition 14 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET).$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1A \in (2^{A_27b}).(\forall V2s \in \\ & \quad (2^{A_27a}).(((\forall V3x \in A_27a.(\forall V4y \in A_27a.(((p (ap \\ & \quad (ap (c_2Ebool_2EIN A_27a) V3x) V2s)) \wedge ((p (ap (ap (c_2Ebool_2EIN \\ & \quad A_27a) V4y) V2s)) \wedge ((ap V0f V3x) = (ap V0f V4y)))) \Rightarrow (V3x = V4y)))))) \wedge \\ & \quad (p (ap (c_2Epred_set_2EFINITE A_27b) V1A))) \Rightarrow (p (ap (c_2Epred_set_2EFINITE \\ & \quad A_27a) (ap (c_2Epred_set_2EGSPEC A_27a A_27a) (\lambda V5x \in A_27a. \\ & \quad (ap (ap (c_2Epair_2E_2C A_27a 2) V5x) (ap (ap c_2Ebool_2E_2F_5C \\ & \quad (ap (ap (c_2Ebool_2EIN A_27a) V5x) V2s)) (ap (ap (c_2Ebool_2EIN \\ & \quad A_27b) (ap V0f V5x)) V1A)))))))))) \end{aligned} \tag{8}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(p (ap (ap (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))) \tag{9}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ & \quad \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1A \in (2^{A_{.27b}}).(((\forall V2x \in \\ & \quad A_{.27a}.(\forall V3y \in A_{.27a}.((ap\ V0f\ V2x) = (ap\ V0f\ V3y)) \Rightarrow (V2x = V3y)))))) \wedge \\ & (p\ (ap\ (c_{.2}Epred_set_2EFINITE\ A_{.27b})\ V1A)) \Rightarrow (p\ (ap\ (c_{.2}Epred_set_2EFINITE \\ & \quad A_{.27a})\ (ap\ (c_{.2}Epred_set_2EGSPEC\ A_{.27a}\ A_{.27a})\ (\lambda V4x \in A_{.27a}. \\ & \quad (ap\ (ap\ (c_{.2}Epair_2E_2C\ A_{.27a}\ 2)\ V4x)\ (ap\ (ap\ (c_{.2}Ebool_2EIN\ A_{.27b}) \\ & \quad (ap\ V0f\ V4x))\ V1A)))))))))) \end{aligned}$$