thm_2Ecardinal_2EHAS__SIZE__CART__UNIV (TMXD1FxdNNK8ASXbiBjqbgkPmhfT6irkkKN)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c_2Ebool_2ET)$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (1)$$

Let $c\_2Epred\_set\_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_set\_2ECARD\ A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})})$$
$$(2)$$

Definition 4 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$
$$(3)$$

Let $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$
$$(4)$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epred\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_set\_2EGSPEC$$
$$A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(5)$$

Definition 10 *We define* c_2Epred_set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 11 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 *We define* c_2Epred_set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$.

Definition 13 *We define* c_2Epred_set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (c\_2Ebool\_2E\_21\ (2$

Definition 14 *We define* c_2Ecardinal_2EHAS__SIZE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1n \in ty\_2Enum\_$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (6)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (7)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 16 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (8)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (9)$$

Definition 17 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(10)$$

Definition 18 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 19 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(11)$$

Definition 20 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap \ (ap \ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E} \ V0t) \ \mathsf{c\_2Ebool\_2E}$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(((ap \ (ap \ c\_2Earithmetic\_2EEXP$$
$$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO})))$$
$$V0n) = (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1}$$
$$\mathsf{c\_2Earithmetic\_2EZERO})))\wedge((ap \ (ap \ c\_2Earithmetic\_2EEXP \ V0n)$$
$$(ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL} \ (ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO}))) =$$
$$V0n)))$$

$$(12)$$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in$$
$$A\_27a.(p \ V0t))\Leftrightarrow(p \ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t))\Leftrightarrow(p \ V0t))\wedge((((p \ V0t)\Rightarrow True)\Leftrightarrow$$
$$True)\wedge(((False \Rightarrow (p \ V0t))\Leftrightarrow True)\wedge((((p \ V0t)\Rightarrow(p \ V0t))\Leftrightarrow True)\wedge((\tag{15}$$
$$(p \ V0t)\Rightarrow False)\Leftrightarrow(\neg(p \ V0t)))))))))$$

Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum.($$
$$(p \ (ap \ (ap \ (\mathsf{c\_2Ecardinal\_2EHAS\_\_SIZE} \ A\_27a) \ (\mathsf{c\_2Epred\_\_set\_2EUNIV}$$
$$A\_27a)) \ V0m))\Rightarrow(p \ (ap \ (ap \ (\mathsf{c\_2Ecardinal\_2EHAS\_\_SIZE} \ A\_27a) \ (\mathsf{c\_2Epred\_\_set\_2EUNIV}$$
$$A\_27a)) \ (ap \ (ap \ c\_2Earithmetic\_2EEXP \ V0m) \ (ap \ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap \ \mathsf{c\_2Earithmetic\_2EBIT1} \ \mathsf{c\_2Earithmetic\_2EZERO}))))))))$$