

# thm\_2Ecardinal\_2EHAS\_\_SIZE\_\_NUMSEG\_\_LE (TMLd1hBQy1if9CmJo1QCqFAXc3XygKBG9qF)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmic\_2EBIT1 V0n))$ .

**Definition 8** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$ .

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then  $(\lambda x.x \in A \wedge P x)$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 V0P))))$ .

**Definition 15** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$ .

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$ .

**Definition 17** We define  $c\_2Earithmic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.V1n$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (7)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (8)$$

**Definition 18** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod V0x V1y))$ .

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (9)$$

Let  $c\_2Epred\_set\_2ECARD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Epred\_set\_2ECARD A\_27a \in (ty\_2Enum\_2Enum^{(2^{A\_27a})}) \quad (10)$$

**Definition 19** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$ .

**Definition 20** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2Epair\_2EABS\_prod V0x V1s))$ .

**Definition 21** We define `c_2Epred_set_2EEMPTY` to be  $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$ .

**Definition 22** We define `c_2Epred_set_2EFINITE` to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).(ap (c_2Ebool_2E_21 (2$

**Definition 23** We define `c_2Ecardinal_2EHAS_SIZE` to be  $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1n \in ty_2Enum.2$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC V0m) = (ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A.27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (15)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap c_2Enum_2ESUC V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)))))) \quad (16)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap (c_2Ecardinal_2EHAS_SIZE ty_2Enum_2Enum) (ap (c_2Epred_set_2EGSPEC ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V1m \in ty_2Enum_2Enum.(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum 2) V1m) (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)))))) V0n))) \quad (17)$$

**Theorem 1**

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap (c_2Ecardinal_2EHAS_SIZE ty_2Enum_2Enum) (ap (c_2Epred_set_2EGSPEC ty_2Enum_2Enum ty_2Enum_2Enum) (\lambda V1m \in ty_2Enum_2Enum.(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum 2) V1m) (ap (ap c_2Earithmetic_2E_3C_3D V1m) V0n)))))) (ap (ap c_2Earithmetic_2E_2B V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))$$