

# thm\_2Ecardinal\_2ELEFT\_\_IMP\_\_EXISTS\_\_THM (TMNLovB2GyVDBmXpPh1SpSbDxHyyJL3j2PN)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p (ap P x))$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))))$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}) P) P))))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p (ap V0P V3x)) \Rightarrow (p V1Q)))) \wedge (((\exists V4x \in A\_27a.(p (ap V0P V4x))) \wedge (p V1Q)) \Leftrightarrow (\exists V5x \in A\_27a.((p (ap V0P V5x)) \wedge (p V1Q)))) \wedge (((p V1Q) \wedge (\exists V6x \in A\_27a.(p (ap V0P V6x)))) \Leftrightarrow (\exists V7x \in A\_27a.((p V1Q) \wedge (p (ap V0P V7x)))))))))) \end{aligned} \tag{4}$$

**Theorem 1**

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0 P \in (2^{A_{27a}}). (\forall V_1 Q \in 2. (((\exists V_2 x \in A_{27a}. (p (ap V_0 P V_2 x))) \Rightarrow (p V_1 Q)) \Leftrightarrow (\forall V_3 x \in A_{27a}. ((p (ap V_0 P V_3 x)) \Rightarrow (p V_1 Q)))))))$$