# thm_2Ecardinal_2ENUM__COUNTABLE (TMQdYEWa8ZdcL9EaGg3V1ZzKfMcQb41eEkZ)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

**Definition 7** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

**Definition 8** *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2ET$)$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 9** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 10** *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

**Definition 11** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

**Definition 12** *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

**Definition 13** *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A}$

**Definition 14** *We define* c_2Ecardinal_2Ecardgeq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_2}$

**Definition 15** *We define* c_2Epred__set_2Ecountable *to be* $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ ($c_2Ebool_2E_3F

Assume the following.

$$True \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( p\ V0t))))))) \tag{3}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardgeq}\ A\_27a\ A\_27b)\ V0s)\ V1t)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1t)\ V0s))))) \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2Ecountable}\ A\_27a)\ V0t)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardgeq}\ ty\_2Enum\_2Enum\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum))\ V0t)))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(p\ (ap\ ( ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27a)\ V0s)\ V0s))) \tag{6}$$

**Theorem 1**

$$(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2Ecountable}\ ty\_2Enum\_2Enum)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum)))$$