

thm_2Ecardinal_2ERIGHT__IMP__EXISTS__THM (TMJ7nRpzUuM2UymEoBLTjYZ2HUgVmriZ2tv)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 4 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 5 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a))))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Assume the following.

$$\text{True} \tag{1}$$

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$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V1t))) \tag{2}$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \tag{3}$$

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$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{4}$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p V0P) \Rightarrow (p (\text{ap } V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\exists V3x \in A_27a. (p (\text{ap } V1Q V3x)))))))) \tag{5}$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V_0 P \in 2. (\forall V_1 Q \in (2^{A_{27a}}). (((p \ V_0 P) \Rightarrow (\exists V_2 x \in A_{27a}. (p \ (ap \ V_1 Q \ V_2 x)))) \Leftrightarrow (\exists V_3 x \in A_{27a}. ((p \ V_0 P) \Rightarrow (p \ (ap \ V_1 Q \ V_3 x))))))))$$