

thm_2Ecardinal_2ESET__SQUARED__CARDEQ__SET (TMG7fY1VQ1j79w8DTL5WV51WpnQkTHeMuUC)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P)))$

Definition 9 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 10 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 11 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a})$

Definition 12 We define $c_2Ecardinal_2Ecardeq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27a})$

Definition 13 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27a})$

Definition 14 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 15 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 16 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 17 We define $c_2E_marker_2E_abbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 18 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2E_pair_2E_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2E_pair_2E_prod\ A0\ A1) \tag{4}$$

Let $c_2Enum_pair_2E_invtri0 : \iota$ be given. Assume the following.

$$c_2Enum_pair_2E_invtri0 \in (((ty_2E_pair_2E_prod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2E_pair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2E_pair_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty_2E_pair_2E_prod\ A.27a\ A.27b)} \tag{6}$$

Definition 19 We define $c_2Enum_pair_2E_invtri$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2E_pair_2ESND\ ty_2Enum_2Enum\ n))$

Let $c_2Enum_pair_2E_tri : \iota$ be given. Assume the following.

$$c_2Enum_pair_2E_tri \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \tag{7}$$

Let $c_2E_arithmic_2E_2D : \iota$ be given. Assume the following.

$$c_2E_arithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{8}$$

Definition 20 We define $c_2Enum_pair_2E_ensnd$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2E_arithmic_2E_2D\ n))$

Let $c_2E_arithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2E_arithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{9}$$

Definition 21 We define $c_2Enum_pair_2E_enfst$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2E_arithmic_2E_2B\ n))$

Definition 22 We define $c_2Enumpair_2Enpair$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (10)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \end{aligned} \quad (11)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (12)$$

Definition 23 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 24 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 25 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum \\ A0\ A1) \end{aligned} \quad (13)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (14)$$

Definition 26 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (15)$$

Definition 27 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eone_2Eone))$

Definition 28 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 29 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 30 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2E_21\ 2)\ t1\ t2))\ V0t)$

Definition 31 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ c_2Eoption_2ENONE)\ V0P)$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Definition 32 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$.

Definition 33 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Eprod\ A_27a\ A_27b)$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (17)$$

Definition 34 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2Eprod\ A_27a\ A_27b)\ x\ y)$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (18)$$

Definition 35 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ s\ t)$.

Definition 36 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 37 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ s\ t)$.

Definition 38 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ s\ t)$.

Definition 39 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$.

Definition 40 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ x\ s)$.

Definition 41 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ s\ t)$.

Definition 42 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27b)\ f\ s)$.

Definition 43 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27a)\ P)$.

Definition 44 We define $c_2Epred_set_2ECROSS$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1Q \in (2^{A_27b}).(ap\ (c_2Epred_set_2Eprod\ A_27a\ A_27b)\ P\ Q)$.

Definition 45 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 46 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_21\ 2)\ s)$.

Definition 47 We define $c_2Eset_relation_2Eantisym$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Definition 48 We define $c_Eset_relation_Ereflexive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 49 We define $c_Eset_relation_Etransitive$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 50 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 51 We define $c_Epred_set_ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (V0s\ V1t))$

Definition 52 We define $c_Eset_relation_ERange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 53 We define $c_Eset_relation_Emaximal_elements$ to be $\lambda A_27a : \iota.\lambda V0xs \in (2^{A_27a}).\lambda V1r \in (2^{A_27a})$

Definition 54 We define $c_Eset_relation_Eupper_bounds$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{A_27b}).\lambda V1r \in (2^{A_27a})$

Definition 55 We define $c_Eset_relation_Echain$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 56 We define $c_Eset_relation_Epartial_order$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{22}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \tag{23}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V1x)))) \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \quad (35)$$

Assume the following.

$$((\neg(True \Leftrightarrow False)) \wedge (\neg(False \Leftrightarrow True))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p (ap V0P V2x)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p (ap V0P V2x)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A_27a. (p (ap V1Q V4x))))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. (((\forall V2x \in A_27a. (p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a. ((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p V0P) \wedge (\forall V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x))) \vee (\exists V4x \in A_27a. (p (ap V1Q V4x)))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((p V0P) \vee (\exists V2x \in A.27a. (p (ap V1Q V2x)))))) \Leftrightarrow (\exists V3x \in A.27a. ((p V0P) \vee (p (ap V1Q V3x)))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\exists V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q)))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \wedge (p V1Q)))))) \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (47)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (49)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x))) \Rightarrow (p V1Q)))))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B)))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge ((p V2C) \vee (p V0A))) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))))) \quad (58)$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2R \in 2.(((p V0P) \Rightarrow ((p V1Q) \wedge (p V2R))) \Leftrightarrow (((p V0P) \Rightarrow (p V1Q)) \wedge ((p V0P) \Rightarrow (p V2R)))))) \quad (59)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0b \in 2.(\forall V1f \in (A_27b^{A_27a}).(\forall V2g \in (A_27b^{A_27a}). \\ & \quad (\forall V3x \in A_27a.((ap (ap (ap (ap (c_2Ebool_2ECOND (A_27b^{A_27a}) \\ & \quad V0b) V1f) V2g) V3x) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V0b) (ap \\ & \quad V1f V3x)) (ap V2g V3x)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1b \in 2.(\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a.((ap V0f (ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & \quad V1b) V2x) V3y) = (ap (ap (ap (c_2Ebool_2ECOND A_27b) V1b) (ap V0f \\ & \quad V2x)) (ap V0f V3y)))))) \end{aligned} \quad (62)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (63)$$

Assume the following.

$$2.(((\forall V0P \in 2.(\forall V1P_{.27} \in 2.(\forall V2Q \in 2.(\forall V3Q_{.27} \in 2.(((p V2Q) \Rightarrow ((p V0P) \Leftrightarrow (p V1P_{.27})) \wedge ((p V1P_{.27}) \Rightarrow ((p V2Q) \Leftrightarrow (p V3Q_{.27})))))) \Rightarrow ((p V0P) \wedge (p V2Q)) \Leftrightarrow ((p V1P_{.27}) \wedge (p V3Q_{.27})))))) \Rightarrow (64)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}.(\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) V1Q) V3x_{.27}) V5y_{.27})))))) \Rightarrow (65)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}.(\exists V1x \in A_{.27a}.(V1x = V0a))) \Rightarrow (66)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))) \Rightarrow (67)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))) \Rightarrow (68)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0P \in ((2^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in A_{.27a}.(\exists V2y \in A_{.27b}.(p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{.27b}^{A_{.27a}}).(\forall V4x \in A_{.27a}.(p (ap (ap V0P V4x) (ap V3f V4x)))))) \Rightarrow (69)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in A_{.27a}.((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) c_{.2Ebool_2ET}) V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap (ap (ap (c_{.2Ebool_2ECOND} A_{.27a}) c_{.2Ebool_2EF}) V2t1) V3t2) = V3t2)))) \Rightarrow (70)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27a)\ V0s)\ V0s))) \quad (71)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). ((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27b)\ V0s)\ V1t)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27b\ A.27a)\ V1t)\ V0s)))))) \quad (72)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). \\ (\forall V2u \in (2^{A.27c}). (((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27b\ A.27c)\ V1t)\ V2u)))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27c)\ V0s)\ V2u)))))) \end{aligned} \quad (73)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27a\ A.27a)\ V0s)\ V0s))) \quad (74)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). \\ (\forall V2u \in (2^{A.27c}). (((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27a\ A.27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27b\ A.27c)\ V1t)\ V2u)))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27a\ A.27c)\ V0s)\ V2u)))))) \end{aligned} \quad (75)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). (((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27a\ A.27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27b\ A.27a)\ V1t)\ V0s)))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27b)\ V0s)\ V1t)))))) \quad (76)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). (((p\ (ap\ (c.2Epred_set_EFINITE\ A.27a)\ V0s)) \wedge (\neg (p\ (ap\ (c.2Epred_set_EFINITE\ A.27b)\ V1t)))))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardleq\ A.27a\ A.27b)\ V0s)\ V1t)))))) \quad (77)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0s1 \in (\\
& \quad \quad 2^{A_27a}).(\forall V1s2 \in (2^{A_27b}).(\forall V2t1 \in (2^{A_27c}). \\
& \quad \quad (\forall V3t2 \in (2^{A_27d}).(((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq \\
& \quad \quad A_27a\ A_27b)\ V0s1)\ V1s2)) \wedge (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ A_27c \\
& \quad \quad A_27d)\ V2t1)\ V3t2)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ (ty_2Epair_2Eprod \\
& \quad \quad A_27a\ A_27c)\ (ty_2Epair_2Eprod\ A_27b\ A_27d))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad \quad A_27a\ A_27c)\ V0s1)\ V2t1))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27b \\
& \quad \quad A_27d)\ V1s2)\ V3t2)))))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A_27a}).(\forall V1t \in (2^{A_27b}).((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq \\
& \quad \quad A_27a\ A_27b)\ V0s)\ V1t)) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a \\
& \quad \quad A_27b)\ V0s)\ V1t))))))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0s1 \in (\\
& \quad \quad 2^{A_27a}).(\forall V1s2 \in (2^{A_27b}).(\forall V2t1 \in (2^{A_27c}). \\
& \quad \quad (\forall V3t2 \in (2^{A_27d}).(((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq \\
& \quad \quad A_27a\ A_27b)\ V0s1)\ V1s2)) \wedge (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardeq\ A_27c \\
& \quad \quad A_27d)\ V2t1)\ V3t2)))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a \\
& \quad \quad A_27c)\ V0s1)\ V2t1)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27b\ A_27d) \\
& \quad \quad V1s2)\ V3t2)))))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0x1 \in (\\
& \quad \quad 2^{A_27a}).(\forall V1x2 \in (2^{A_27b}).(\forall V2y1 \in (2^{A_27c}). \\
& \quad \quad (\forall V3y2 \in (2^{A_27d}).(((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\
& \quad \quad A_27a\ A_27b)\ V0x1)\ V1x2)) \wedge (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27c \\
& \quad \quad A_27d)\ V2y1)\ V3y2)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Epair_2Eprod \\
& \quad \quad A_27a\ A_27c)\ (ty_2Epair_2Eprod\ A_27b\ A_27d))\ (ap\ (ap\ (c_2Epred_set_2ECROSS \\
& \quad \quad A_27a\ A_27c)\ V0x1)\ V2y1))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27b \\
& \quad \quad A_27d)\ V1x2)\ V3y2)))))))))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A_27a}).(\forall V1y \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0x)\ V1y)) \Rightarrow \\
& \quad (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a\ A_27a)\ V0x)\ V1y))))))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\ \forall V0s \in (2^{A_{.27a}}).(\forall V1t \in (2^{A_{.27b}}).((\neg(p\ (ap\ (ap\ (\\ c_2Ecardinal_2Ecardleq\ A_{.27b}\ A_{.27a})\ V1t)\ V0s))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ A_{.27a}\ A_{.27b})\ V0s)\ V1t)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_{.27a} \\ A_{.27b})\ V0s)\ V1t)))))))) \end{aligned} \quad (83)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0A \in (2^{A_{.27a}}).(\forall V1B \in \\ (2^{A_{.27a}}).((ap\ (ap\ (c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27a})\ (ap\ (ap \\ (c_2Epred_set_2EUNION\ A_{.27a})\ V0A)\ V1B))\ (ap\ (ap\ (c_2Epred_set_2EUNION \\ A_{.27a}\ V0A)\ V1B))) = (ap\ (ap\ (c_2Epred_set_2EUNION\ (ty_2Epair_2Eprod \\ A_{.27a}\ A_{.27a}))\ (ap\ (ap\ (c_2Epred_set_2EUNION\ (ty_2Epair_2Eprod \\ A_{.27a}\ A_{.27a}))\ (ap\ (ap\ (c_2Epred_set_2EUNION\ (ty_2Epair_2Eprod \\ A_{.27a}\ A_{.27a}))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27a})\ V0A) \\ V0A)))\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27a})\ V0A)\ V1B))))\ (\\ ap\ (ap\ (c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27a})\ V1B)\ V0A)))\ (ap\ (ap\ (\\ c_2Epred_set_2ECROSS\ A_{.27a}\ A_{.27a})\ V1B)\ V1B)))))) \end{aligned} \quad (84)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap\ (c_2Ecombin_2EI \\ A_{.27a})\ V0x) = V0x)) \quad (85)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\\ (ap\ c_2Enumpair_2Enfst\ (ap\ (ap\ c_2Enumpair_2Enpair\ V0x)\ V1y))) = \\ V0x)) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(\\ (ap\ c_2Enumpair_2Ensnd\ (ap\ (ap\ c_2Enumpair_2Enpair\ V0x)\ V1y))) = \\ V1y)) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Enumpair_2Enpair\ (ap \\ c_2Enumpair_2Enfst\ V0n))\ (ap\ c_2Enumpair_2Ensnd\ V0n)) = V0n)) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} (\forall V0x1 \in ty_2Enum_2Enum.(\forall V1y1 \in ty_2Enum_2Enum. \\ (\forall V2x2 \in ty_2Enum_2Enum.(\forall V3y2 \in ty_2Enum_2Enum. \\ (((ap\ (ap\ c_2Enumpair_2Enpair\ V0x1)\ V1y1) = (ap\ (ap\ c_2Enumpair_2Enpair \\ V2x2)\ V3y2))) \Leftrightarrow ((V0x1 = V2x2) \wedge (V1y1 = V3y2)))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\
& (2^{(ty_2Eoption_2Eoption\ A_27a)}). (((\forall V2x \in A_27a. ((p \\
& (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2x)))))) \wedge \\
& ((\forall V3x \in A_27a. (\neg (p\ (ap\ V0P\ V3x)))) \Rightarrow (p\ (ap\ V1Q\ (c_2Eoption_2ENONE \\
& A_27a)))))) \Rightarrow (p\ (ap\ V1Q\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0P))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Eoption_2ESOME\ A_27a) \\
& (\lambda V0x \in A_27a.c_2Ebool_2EF)) = (c_2Eoption_2ENONE\ A_27a))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0y \in A_27a. (((ap\ (c_2Eoption_2ESOME \\
& A_27a)\ (\lambda V1x \in A_27a. (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1x)\ V0y))) = \\
& (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0y)) \wedge ((ap\ (c_2Eoption_2ESOME \\
& A_27a)\ (\lambda V2x \in A_27a. (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V0y)\ V2x))) = \\
& (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0y))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\exists V1q \in A_27a. \\
& (\exists V2r \in A_27b. (V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\
& V1q)\ V2r))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).(\exists V1q \in A.27a. \\ & (\exists V2r \in A.27b.(V0x = (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b) \\ & V1q)\ V2r)))))) \end{aligned} \quad (97)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A.27a\ A.27b).((ap\ (ap\ (c.2Epair_2E_2C \\ & A.27a\ A.27b)\ (ap\ (c.2Epair_2EFST\ A.27a\ A.27b)\ V0x))\ (ap\ (c.2Epair_2ESND \\ & A.27a\ A.27b)\ V0x)) = V0x)) \end{aligned} \quad (98)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair_2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (99)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair_2ESND\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (100)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}).(\forall V1x \in \\ & A.27a.(\forall V2y \in A.27b.((ap\ (ap\ (c.2Epair_2EUNCURRY\ A.27a \\ & A.27b\ A.27c)\ V0f)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V1x)\ V2y)) = \\ & (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (101)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}).((\exists V1p \in \\ & (ty_2Epair_2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\exists V2p_1 \in \\ & A.27a.(\exists V3p_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair_2E_2C \\ & A.27a\ A.27b)\ V2p_1)\ V3p_2))))))) \end{aligned} \quad (102)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}).((\forall V1p \in \\ & (ty_2Epair_2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ & A.27a.(\forall V3p_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair_2E_2C \\ & A.27a\ A.27b)\ V2p_1)\ V3p_2))))))) \end{aligned} \quad (103)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0x \in A.27b. (\forall V1y \in A.27c. (\forall V2f \in \\ & ((A.27a^{A.27c})^{A.27b}). ((ap\ (ap\ (c.2Epair.2Epair_CASE\ A.27a\ A.27b \\ & A.27c)\ (ap\ (ap\ (c.2Epair.2E.2C\ A.27b\ A.27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (104)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ & (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (105)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in ((ty.2Epair.2Eprod\ A.27a\ 2)^{A.27b}). (\forall V1v \in \\ & A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1v)\ (ap\ (c.2Epred_set.2EGSPEC \\ & A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b. ((ap\ (ap\ (c.2Epair.2E.2C \\ & A.27a\ 2)\ V1v)\ c.2Ebool.2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (106)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EEMPTY\ A.27a)))))) \quad (107)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). ((\exists V1x \in \\ & A.27a. (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s))) \Leftrightarrow (\neg (V0s = (c.2Epred_set.2EEMPTY \\ & A.27a)))))) \end{aligned} \quad (108)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EUNIV\ A.27a)))) \quad (109)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ & (2^{A.27a}). (\forall V2u \in (2^{A.27a}). (((p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET \\ & A.27a)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V1t \\ & V2u)))) \Rightarrow (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V0s)\ V2u)))))) \end{aligned} \quad (110)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V0s)\ V0s))) \quad (111)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((p (ap (ap (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V1t)) \wedge \\ & (p (ap (ap (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s))) \Rightarrow (V0s = V1t)))) \end{aligned} \quad (112)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ \\ & V2x)\ (ap (ap (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p (ap \quad (113) \\ & (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p (ap (ap (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2u \in (2^{A_27a}). ((ap (ap (c_2Epred_set_2EUNION \\ & A_27a)\ V0s)\ (ap (ap (c_2Epred_set_2EUNION\ A_27a)\ V1t)\ V2u)) = (\\ & ap (ap (c_2Epred_set_2EUNION\ A_27a)\ (ap (ap (c_2Epred_set_2EUNION \\ & A_27a)\ V0s)\ V1t))\ V2u)))))) \end{aligned} \quad (114)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (p (ap (ap (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ (ap (\\ & ap (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t)))))) \wedge (\forall V2s \in \quad (115) \\ & (2^{A_27a}). (\forall V3t \in (2^{A_27a}). (p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27a)\ V2s)\ (ap (ap (c_2Epred_set_2EUNION\ A_27a)\ V3t)\ V2s)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2u \in (2^{A_27a}). ((p (ap (ap (c_2Epred_set_2ESUBSET \\ & A_27a)\ (ap (ap (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))\ V2u)) \Leftrightarrow \\ & ((p (ap (ap (c_2Epred_set_2ESUBSET\ A_27a)\ V0s)\ V2u)) \wedge (p (ap (ap \\ & (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V2u)))))) \end{aligned} \quad (116)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ \\ & V2x)\ (ap (ap (c_2Epred_set_2EINTER\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p (ap \quad (117) \\ & (ap (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \wedge (p (ap (ap (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). (\forall V2u \in (2^{A_27a}). (((p (ap (ap (c_2Epred_set_2EDISJOINT \\
& A_27a) (ap (ap (c_2Epred_set_2EUNION\ A_27a) V0s) V1t)) V2u)) \Leftrightarrow \\
& ((p (ap (ap (c_2Epred_set_2EDISJOINT\ A_27a) V0s) V2u)) \wedge (p (ap \\
& (ap (c_2Epred_set_2EDISJOINT\ A_27a) V1t) V2u)))) \wedge ((p (ap (ap \\
& (c_2Epred_set_2EDISJOINT\ A_27a) V2u) (ap (ap (c_2Epred_set_2EUNION \\
& A_27a) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c_2Epred_set_2EDISJOINT\ A_27a) \\
& V0s) V2u)) \wedge (p (ap (ap (c_2Epred_set_2EDISJOINT\ A_27a) V1t) V2u))))))))) \\
& \hspace{15em} (118)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& (2^{A_27a}). (\forall V2x \in A_27a. ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\
& V2x) (ap (ap (c_2Epred_set_2EDIFF\ A_27a) V0s) V1t))) \Leftrightarrow ((p (ap (\\
& ap (c_2Ebool_2EIN\ A_27a) V2x) V0s)) \wedge (\neg (p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V2x) V1t))))))))) \\
& \hspace{15em} (119)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\
& A_27a. (\forall V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN\ A_27a) \\
& V0x) (ap (ap (c_2Epred_set_2EINSERT\ A_27a) V1y) V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p (ap (ap (c_2Ebool_2EIN\ A_27a) V0x) V2s))))))))) \\
& \hspace{15em} (120)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& ((p (ap (ap (c_2Ebool_2EIN\ A_27b) V0y) (ap (ap (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b) V2f) V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap V2f V3x)) \wedge \\
& (p (ap (ap (c_2Ebool_2EIN\ A_27a) V3x) V1s))))))))) \\
& \hspace{15em} (121)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN \\
& A_27a) V0x) V1s)) \Rightarrow (\forall V2f \in (A_27b^{A_27a}). (p (ap (ap (c_2Ebool_2EIN \\
& A_27b) (ap V2f V0x)) (ap (ap (c_2Epred_set_2EIMAGE\ A_27a\ A_27b) \\
& V2f) V1s))))))))) \\
& \hspace{15em} (122)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (p (ap (ap (\\
& ap (c_2Epred_set_2ESURJ\ A_27a\ A_27b) V0f) V1s) (ap (ap (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b) V0f) V1s)))))) \\
& \hspace{15em} (123)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (c.2Epred_set.2EEMPTY\ A.27a))) \quad (124)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ & (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)))))) \end{aligned} \quad (125)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ & (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EUNION \\ & A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V0s)) \wedge \\ & (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1t)))))) \end{aligned} \quad (126)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (A.27b^{A.27a}). ((\forall V1x \in A.27a. (\forall V2y \in \\ & A.27a. (((ap\ V0f\ V1x) = (ap\ V0f\ V2y)) \Leftrightarrow (V1x = V2y)))) \Rightarrow (\forall V3s \in \\ & (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27b)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE \\ & A.27a\ A.27b)\ V0f)\ V3s))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ \\ & V3s)))))) \end{aligned} \quad (127)$$

Assume the following.

$$(\neg (p\ (ap\ (c.2Epred_set.2EFINITE\ ty.2Enum.2Enum)\ (c.2Epred_set.2EUNIV\ ty.2Enum.2Enum)))) \quad (128)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1sos \in \\ & (2^{(2^{A.27a})}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (ap\ (c.2Epred_set.2EBIGUNION \\ & A.27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & A.27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (129)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0X \in (2^{A.27a}). (\forall V1P \in \\ & (2^{(2^{A.27a})}). ((p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ (ap \\ & (c.2Epred_set.2EBIGUNION\ A.27a)\ V1P))\ V0X))) \Leftrightarrow (\forall V2Y \in (\\ & 2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (2^{A.27a})\ V2Y)\ V1P))) \Rightarrow (p\ (\\ & ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V2Y)\ V0X)))))) \end{aligned} \quad (130)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A-27a})}).((\\ & p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (c_2Epred_set_2EBIGUNION \\ & A_27a)\ V0P))) \Leftrightarrow ((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{A-27a})\ V0P)) \wedge \\ & (\forall V1s \in (2^{A-27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A-27a}) \\ & V1s)\ V0P))) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)))))) \\ & \end{aligned} \tag{131}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A-27a}).(\forall V1P \in \\ & (2^{(2^{A-27a})}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A-27a})\ V0x)\ V1P))) \Rightarrow \\ & (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\ & A_27a)\ V1P)))))) \\ & \end{aligned} \tag{132}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{A-27a}).(\forall V1Q \in (2^{A-27b}).(\forall V2x \in \\ & (ty_2Epair_2Eprod\ A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ & A_27a\ A_27b)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2ECROSS\ A_27a\ A_27b) \\ & V0P)\ V1Q))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ (ap\ (c_2Epair_2EFST \\ & A_27a\ A_27b)\ V2x))\ V0P)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V2x))\ V1Q)))))) \\ & \end{aligned} \tag{133}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}).((\neg(p\ (\\ & ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s))) \Leftrightarrow (\exists V1f \in (A_27a^{ty_2Enum_2Enum}). \\ & (p\ (ap\ (ap\ (c_2Epred_set_2EINJ\ ty_2Enum_2Enum\ A_27a)\ V1f) \\ & (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)\ V0s)))))) \\ & \end{aligned} \tag{134}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{135}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{136}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\ & \end{aligned} \tag{137}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\ & \end{aligned} \tag{138}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (139)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (140)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \end{aligned} \quad (141)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (142)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (143)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (144)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in \\ & 2.(((p V0p) \Leftrightarrow (p (ap (ap (ap (c_2Ebool.2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\ & (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\ & (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(\\ & p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (145)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (146)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (147)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (148)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (149)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (150)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0r \in (2^{(ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a})}). \\ & (\forall V1s \in (2^{A_{.27a}}).(((\neg(V1s = (c_2Epred_set_2EEMPTY \ A_{.27a}))) \wedge \\ & ((p \ (ap \ (ap \ (c_2Eset_relation_2Epartial_order \ A_{.27a}) \ V0r) \ V1s)) \wedge \\ & (\forall V2t \in (2^{A_{.27a}}).((p \ (ap \ (ap \ (c_2Eset_relation_2Echain \\ & A_{.27a}) \ V2t) \ V0r)) \Rightarrow (\neg((ap \ (ap \ (c_2Eset_relation_2Eupper_bounds \\ & A_{.27a} \ A_{.27a}) \ V2t) \ V0r) = (c_2Epred_set_2EEMPTY \ A_{.27a})))))) \Rightarrow \\ & (\exists V3x \in A_{.27a}.(p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_{.27a}) \ V3x) \ (ap \ (ap \\ & (c_2Eset_relation_2Emaximal_elements \ A_{.27a}) \ V1s) \ V0r)))))) \end{aligned} \quad (151)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}).((\neg(p \ (\\ & ap \ (c_2Epred_set_2EFINITE \ A_{.27a}) \ V0s))) \Rightarrow (p \ (ap \ (ap \ (c_2Ecardinal_2Ecardeq \\ & (ty_2Epair_2Eprod \ A_{.27a} \ A_{.27a}) \ A_{.27a}) \ (ap \ (ap \ (c_2Epred_set_2ECROSS \\ & A_{.27a} \ A_{.27a}) \ V0s) \ V0s)))))) \end{aligned}$$