

thm_2Ecardinal_2ESURJECTIVE__IFF__INJECTIVE (TMUv5odtxQnC2nKaFekc6Z9cv7UBLiwhw67)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 9 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}})$$
(3)

Definition 12 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in$

Definition 13 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum$$
(4)

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Epred_set_2ECARD A_27a \in (ty_2Eenum_2Eenum^{(2^{A_27a})})$$
(5)

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 15 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2F).$

Definition 17 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True$$
(6)

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t)))$$
(7)

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned}$$
(8)

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned}$$
(9)

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True))$$
(10)

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (& \\ \forall V0s \in (2^{A_27a}). (\forall V1t \in (2^{A_27b}). (\forall V2f \in & \\ (A_27b^{A_27a}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \wedge & \\ ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27b)\ V1t)) \wedge ((ap\ (c_2Epred_set_2ECARD & \\ A_27a)\ V0s) = (ap\ (c_2Epred_set_2ECARD\ A_27b)\ V1t)) \wedge (p\ (ap\ (ap & \\ (c_2Epred_set_2ESUBSET\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE & \\ A_27a\ A_27b)\ V2f)\ V0s))\ V1t)))) \Rightarrow ((\forall V3y \in A_27b. ((p\ (ap\ (& \\ ap\ (c_2Ebool_2EIN\ A_27b)\ V3y)\ V1t)) \Rightarrow (\exists V4x \in A_27a. ((p\ (ap & \\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4x)\ V0s)) \wedge ((ap\ V2f\ V4x) = V3y)))))) \Leftrightarrow (& \\ \forall V5x \in A_27a. (\forall V6y \in A_27a. (((p\ (ap\ (ap\ (c_2Ebool_2EIN & \\ A_27a)\ V5x)\ V0s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V6y)\ V0s)) \wedge & \\ ((ap\ V2f\ V5x) = (ap\ V2f\ V6y)))) \Rightarrow (V5x = V6y)))))) \quad (15) \end{aligned}$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1f \in & \\ (A_27a^{A_27a}). (((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \wedge & \\ (p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE & \\ A_27a\ A_27a)\ V1f)\ V0s))\ V0s)) \Rightarrow ((\forall V2y \in A_27a. ((p\ (ap\ (ap & \\ (c_2Ebool_2EIN\ A_27a)\ V2y)\ V0s)) \Rightarrow (\exists V3x \in A_27a. ((p\ (ap\ (& \\ ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V0s)) \wedge ((ap\ V1f\ V3x) = V2y)))))) \Leftrightarrow (\forall V4x \in & \\ A_27a. (\forall V5y \in A_27a. (((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V4x & \\ V0s)) \wedge ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V5y)\ V0s)) \wedge ((ap\ V1f\ V4x) = & \\ (ap\ V1f\ V5y)))) \Rightarrow (V4x = V5y)))))) \quad \end{aligned}$$