

thm_2Ecardinal_2Ecardleq__REFL (TMN- qAW2KJMfW6jszHm9nCTEqHKh1A2rraYv)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a})) (\lambda V1P \in 2. V1P)) (\lambda V2P \in 2. V2P)))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\lambda V1t \in 2. V1t)) (\lambda V2t \in 2. V2t)))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\lambda x. x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A_27a) (\lambda V1P \in 2^{A_27a}. V1P))))$

Definition 10 We define `c_2Ebool_2E_2IN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (\text{ap } V1f V0x))))$

Definition 11 We define `c_2Epred__set_2EINJ` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (\text{ap } V1s (\text{ap } (\text{c_2Epred_set_2EINJ } A_27a A_27b) V0f)))$

Definition 12 We define `c_2Ecardinal_2Ecardleq` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s1 \in (2^{A_27a}). \lambda V1s2 \in (2^{A_27b}). (\text{ap } V1s2 (\text{ap } (\text{c_2Ecardleq } A_27a A_27b) V0s1)))$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow \neg(p \ V0t)))))) \end{aligned} \tag{2}$$

Assume the following.

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p (\text{ap } (\text{c_2Epred_set_2EINJ } A_27a A_27a) (\lambda V1x \in A_27a. V1x)) V0s) \Rightarrow \\ & (\text{ap } (\text{c_2Epred_set_2EINJ } A_27a A_27a) (\lambda V1x \in A_27a. V1x)) V0s)) \end{aligned} \tag{3}$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). (p (ap (ap (c_2Ecardinal_2Ecardleq A_{27a} A_{27a}) V0s) V0s)))$$