

# thm\_2Ecardinal\_2Ecardlt\_REFL (TMSvVWD- MJmW68JxkJqc5C5FcyKTAxV8c9Qv)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A\_27a}))))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V0t \in 2. V0t)$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_27E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t)) \text{c\_2Ebool\_2E\_2F}))$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2)) (\lambda V2t \in 2. V2t)))$

**Definition 8** We define `c_2Ebool_2E_2IN` to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (\text{ap } (V1f } V0x))))$

**Definition 9** We define `c_2Epred_set_2E_2INJ` to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

**Definition 10** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define `c_2Ebool_2E_3F` to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40 } A\_27a))))$

**Definition 12** We define `c_2Ecardinal_2Ecardleq` to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0s1 \in (2^{A\_27a}). \lambda V1s2 \in (2^{A\_27b})$

Assume the following.

$$\text{True} \tag{1}$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (4)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). (p (ap (ap (c_{2Ecardinal\_2Ecardleq} A_{27a} A_{27a}) V0s) V0s))) \quad (5)$$

**Theorem 1**

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0s \in (2^{A_{27a}}). (\neg(\neg(p (ap (ap (c_{2Ecardinal\_2Ecardleq} A_{27a} A_{27a}) V0s) V0s))))))$$