

thm_2Ecardinal_2Eeq_c
(TMXmLq5ueGAHq6Fqa8vRCNVjTXjJidFZrap)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Definition 5 We define `c_2Ebool_2E_2IN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a})) P) V0P)))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) V0t1) V1t2))$

Definition 9 We define `c_2Epred_set_2ESURJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V0f V1s)$

Definition 10 We define `c_2Epred_set_2EINJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V0f V1s)$

Definition 11 We define `c_2Epred_set_2EBIJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}). (\text{ap } V0f V1s)$

Definition 12 We define `c_2Ecardinal_2Ecardeq` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0s1 \in (2^{A-27a}). \lambda V1s2 \in (2^{A-27a}). (\text{ap } V0s1 V1s2)$

Definition 13 We define `c_2Ebool_2E_3F_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Ebool_2E_2F_5C } A) P) V0P)))$

Definition 14 We define `c_2Epred_set_2EFUNSET` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0P \in (2^{A-27a}). \lambda V1Q \in (2^{A-27a}). (\text{ap } V0P V1Q)$

Definition 15 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 16 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) V0t1) V1t2))$

Definition 17 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) (\text{c_2Ebool_2E_2F}))))$

Assume the following.

$$True \quad (1)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (2)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (4)$$

Assume the following.

$$(\forall V0t \in 2. (((True) \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (5)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (6)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (7)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Rightarrow (V1y = V0x)))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((True) \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False) \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))) \quad (9)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x)))))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1P \in (2^{A.27a}). (\forall V2Q \in \\ & \quad (2^{A.27b}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\ & \quad (c.2Epred_set.2EFUNSET\ A.27a\ A.27b)\ V1P)\ V2Q))) \Leftrightarrow (\forall V3x \in \\ & \quad A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V3x)\ V1P)) \Rightarrow (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad A.27b)\ (ap\ V0f\ V3x))\ V2Q))))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1s \in (2^{A.27a}). (\forall V2t \in \\ & \quad (2^{A.27b}). ((p\ (ap\ (ap\ (ap\ (c.2Epred_set.2EBIJ\ A.27a\ A.27b)\ V0f)\ \\ & \quad V1s)\ V2t)) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ (A.27b^{A.27a})\ V0f)\ (ap\ (ap \\ & \quad (c.2Epred_set.2EFUNSET\ A.27a\ A.27b)\ V1s)\ V2t))) \wedge (\forall V3y \in \\ & \quad A.27b. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27b)\ V3y)\ V2t)) \Rightarrow (p\ (ap\ (c.2Ebool.2E.3F.21 \\ & \quad A.27a)\ (\lambda V4x \in A.27a. (ap\ (ap\ c.2Ebool.2E.2F.5C\ (ap\ (ap\ (c.2Ebool.2EIN \\ & \quad A.27a)\ V4x)\ V1s))\ (ap\ (ap\ (c.2Emin.2E.3D\ A.27b)\ V3y)\ (ap\ V0f\ V4x))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{23}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& \forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). ((p \ (ap \ (ap \ (c.2Ecardinal_2Ecardeq \\
& A.27a \ A.27b) \ V0s) \ V1t))) \Leftrightarrow (\exists V2f \in (A.27b^{A.27a}). ((\forall V3x \in \\
& A.27a. ((p \ (ap \ (ap \ (c.2Ebool_2EIN \ A.27a) \ V3x) \ V0s)) \Rightarrow (p \ (ap \ (ap \ (c.2Ebool_2EIN \\
& A.27b) \ (ap \ V2f \ V3x)) \ V1t)))) \wedge (\forall V4y \in A.27b. ((p \ (ap \ (ap \ (c.2Ebool_2EIN \\
& A.27b) \ V4y) \ V1t)) \Rightarrow (p \ (ap \ (c.2Ebool_2E_3F_21 \ A.27a) \ (\lambda V5x \in A.27a. \\
& (ap \ (ap \ c.2Ebool_2E_2F_5C \ (ap \ (ap \ (c.2Ebool_2EIN \ A.27a) \ V5x) \ V0s)) \\
& (ap \ (ap \ (c.2Emin_2E_3D \ A.27b) \ (ap \ V2f \ V5x)) \ V4y))))))))))
\end{aligned}$$