thm_2Ecombin_2EASSOC__SYM (TMR-JyqZvNdBHQXFXrk7HaDzQYoA8NyNjZS4)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ecombin_2EASSOC *to be* $\lambda A\_27a : \iota.\lambda V0f \in ((A\_27a^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (1)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ecombin\_2EASSOC}\ A\_27a)\ V0f)) \Leftrightarrow (\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x)\ V2y))\ V3z) = (ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z))))))))$$