

thm\_2Ecombin\_2EGEN\_\_LET\_\_RAND  
(TMaPUk-  
wVMJJq31DLNGLCXWEBZnEHJkUX6hC)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0f \in (A.27b^{A.27a}).(\lambda V1x \in A.27a$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.27a}$

**Definition 5** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in (A.27b^{A.27c}).\lambda V1g$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap (ap (c\_2Ebool\_2ELET A.27a A.27b) V0f) V1x) = (ap V0f V1x)))) \tag{2}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{3}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c.nonempty A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}).(\forall V2x \in A.27c.((ap (ap (ap (c\_2Ecombin\_2Eo A.27c A.27b A.27a) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \tag{4}$$

**Theorem 1**

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c.nonempty A.27c \Rightarrow (\forall V0P \in (A.27a^{A.27b}).(\forall V1f \in (A.27b^{A.27c}).(\forall V2v \in A.27c.((ap V0P (ap (ap (c\_2Ebool\_2ELET A.27c A.27b) V1f) V2v)) = (ap (ap (c\_2Ebool\_2ELET A.27c A.27a) (ap (ap (c\_2Ecombin\_2Eo A.27c A.27a A.27b) V0P) V1f)) V2v))))))$$