

# thm\_2Ecombin\_2EI\_THM (TMTsVPeEWhmp- WNwTHKcDci8o5z3Sfg6ykix)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ecombin_2EK` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

**Definition 3** We define `c_2Ecombin_2ES` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

**Definition 4** We define `c_2Ecombin_2EI` to be  $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

**Definition 5** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow ( \\ \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap (ap (c_2Ecombin_2EK \\ A_27a A_27b) V0x) V1y) = V0x))) \end{aligned} \quad (1)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ nonempty A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).(\forall V1g \in \\ (A_27b^{A_27a}).(\forall V2x \in A_27a.((ap (ap (ap (c_2Ecombin_2ES \\ A_27a A_27b A_27c) V0f) V1g) V2x) = (ap (ap V0f V2x) (ap V1g V2x)))))) \end{aligned} \quad (2)$$

**Theorem 1**

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x))$$