

thm_2Ecombin_2EK_o_THM (TML- RtKVkXun2G8JSTUPaaGSimcA53fP8BUU)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (P \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g$

Definition 8 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{2}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{3}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{4}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27b^{A.27a}).((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A.27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1g \in (A.27a^{A.27c}). \\ (\forall V2x \in A.27c.((ap\ (ap\ (ap\ (c.2Ecombin.2Eo\ A.27c\ A.27b\ A.27a) \\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (ap\ (c.2Ecombin.2EK \\ A.27a\ A.27b)\ V0x)\ V1y) = V0x))) \end{aligned} \quad (7)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty\ A.27e \Rightarrow \forall A.27f.nonempty\ A.27f \Rightarrow ((\forall V0f \in (A.27b^{A.27a}). \\ (\forall V1v \in A.27c.((ap\ (ap\ (c.2Ecombin.2Eo\ A.27a\ A.27c\ A.27b) \\ (ap\ (c.2Ecombin.2EK\ A.27c\ A.27b)\ V1v))\ V0f) = (ap\ (c.2Ecombin.2EK \\ A.27c\ A.27a)\ V1v)))) \wedge (\forall V2f \in (A.27e^{A.27d}).(\forall V3v \in \\ A.27d.((ap\ (ap\ (c.2Ecombin.2Eo\ A.27f\ A.27e\ A.27d)\ V2f)\ (ap\ (c.2Ecombin.2EK \\ A.27d\ A.27f)\ V3v)) = (ap\ (c.2Ecombin.2EK\ A.27e\ A.27f)\ (ap\ V2f\ V3v)))))) \end{aligned}$$