

thm_2Ecombin_2ELET_FORALL_ELIM (TMHPknYJ7wqW3b9zoemjaNSafvi8SDGaebe)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in (A_27b^{A_27a}). (\lambda V1x \in A_27a$

Definition 4 We define c_2Ebool_2EET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27$

Definition 6 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 9 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}$

Definition 10 We define $c_2Ecombin_2EC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_2$

Definition 11 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1$

Definition 12 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Ebool_2ELET\ A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \quad (3)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \quad (4)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27a^{A_27c}).(\forall V2x \in A_27c.((ap\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (5)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in (2^{A_27a}).(\forall V1v \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2ELET\ A_27a\ 2)\ V0f)\ V1v)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ 2\ 2)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ (2^2)\ 2)\ c_2Emin_2E_3D_3D_3E)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ 2\ 2)\ c_2Emarker_2EAbbrev)\ (ap\ (ap\ (c_2Ecombin_2EC\ A_27a\ A_27a\ 2)\ (c_2Emin_2E_3D\ A_27a))\ V1v))))))\ V0f))))))$$