thm_2Ecomparison_2Ebool__cmp__def (TMY4fNp6TUXGfCsmzWYTqKN24ZL5EiyXGVH)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Let $c\_2EternaryComparisons\_2Ebool\_\_compare : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Ebool\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^2)^2) \tag{5}$$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$(((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2ET})\ \mathsf{c\_2Ebool\_2ET}) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_c$$
$$\mathsf{c\_2Ebool\_2EF})\ \mathsf{c\_2Ebool\_2EF}) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2ET})\ \mathsf{c\_2Ebool\_2EF}) = c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2EF})\ \mathsf{c\_2Ebool\_2ET}) = c\_2EternaryComparisons\_2ELESS)))) \tag{6}$$

Theorem 1

$$(((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2ET})\ \mathsf{c\_2Ebool\_2ET}) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_c$$
$$\mathsf{c\_2Ebool\_2EF})\ \mathsf{c\_2Ebool\_2EF}) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2ET})\ \mathsf{c\_2Ebool\_2EF}) = c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ c\_2EternaryComparisons\_2Ebool\_\_compare\ \mathsf{c\_2Ebool\_2EF})\ \mathsf{c\_2Ebool\_2ET}) = c\_2EternaryComparisons\_2ELESS))))$$