thm_2Ecomparison_2Echar__cmp__antisym (TMaUjGfpJ64fx6nW2KzasqD87T9e9yDFygM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Estring\_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{2}$$

Let $c\_2Estring\_2EORD : \iota$ be given. Assume the following.

$$c\_2Estring\_2EORD \in (ty\_2Enum\_2Enum^{ty\_2Estring\_2Echar}) \tag{3}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{5}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{6}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{9}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{10}$$

Definition 13 *We define* c_2EternaryComparisons_2Enum__compare *to be* $\lambda V0n1 \in ty\_2Enum\_2Enum.\lambda V1$

Definition 14 *We define* c_2EternaryComparisons_2Echar__compare *to be* $\lambda V0c1 \in ty\_2Estring\_2Echar.\lambda V$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$(\forall V0c1 \in ty\_2Estring\_2Echar.(\forall V1c2 \in ty\_2Estring\_2Echar.((ap\ (ap\ \mathsf{c\_2EternaryComparisons\_2Echar\_\_compare}\ V0c1)\ V1c2) = (ap\ (ap\ \mathsf{c\_2EternaryComparisons\_2Enum\_\_compare}\ (ap\ c\_2Estring\_2EORD\ V0c1))\ (ap\ c\_2Estring\_2EORD\ V1c2))))) \tag{13}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(\\ ((ap\ (ap\ \mathsf{c\_2EternaryComparisons\_2Enum\_\_compare}\ V0x)\ V1y) = c\_2EternaryComparisons\_2EEQUAL)\Leftrightarrow \\ (V0x = V1y)))) \tag{14}$$

Assume the following.

$$(\forall V0a \in ty\_2Estring\_2Echar.(\forall V1a\_27 \in ty\_2Estring\_2Echar. \\ (((ap\ c\_2Estring\_2EORD\ V0a) = (ap\ c\_2Estring\_2EORD\ V1a\_27))\Leftrightarrow(\\ V0a = V1a\_27)))) \tag{15}$$

Theorem 1

$$(\forall V0x \in ty\_2Estring\_2Echar.(\forall V1y \in ty\_2Estring\_2Echar. \\ (((ap\ (ap\ \mathsf{c\_2EternaryComparisons\_2Echar\_\_compare}\ V0x)\ V1y) = \\ c\_2EternaryComparisons\_2EEQUAL)\Leftrightarrow(V0x = V1y))))$$