thm_2Ecomparison_2Eoption__cmp__cong (TMNiAV1p2YXPKtgxwVXUKBQg1tToEnTTRGG)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{5}$$

Let $c\_2EternaryComparisons\_2Eoption\_\_compare : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparis$$
$$A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Eoption\_2Eoption\ A\_27b)})^{(ty\_2Eoption\_2Eoption\ }$$
(6)

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{7}$$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{8}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{9}$$

Definition 9 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{10}$$

Definition 10 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{15}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{16}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{18}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{19}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V1v0 \in A\_27b.(\forall V2v3 \in A\_27a.(\forall V3v1 \in A\_27a. (\forall V4v2 \in A\_27b.(((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_ \ A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1v0)) = c\_2EternaryComparisons\_2ELESS) \wedge (((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V0c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2v3))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)) = c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_com \ A\_27a\ A\_27b)\ V0c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3v1))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V4v2)) = (ap\ (ap\ V0c\ V3v1)\ V4v2)))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (((2^{(ty\_2Eoption\_2Eoption\ A\_27b)})^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{((ty\_2EternaryComparisons\_2Eordering^{A\_27b}} (((\forall V1c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (p\ (ap\ (ap\ (ap\ V0P\ V1c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \wedge ((\forall V2c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V3v0 \in A\_27b.(p\ (ap\ (ap\ (ap\ V0P\ V2c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V3v0))))) \wedge ((\forall V4c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}).(\forall V5v3 \in A\_27a.(p\ (ap\ (ap\ (ap\ V0P\ V4c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5v3))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))))) \wedge (\forall V6c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V7v1 \in A\_27a.(\forall V8v2 \in A\_27b.(p\ (ap\ (ap\ (ap\ V0P\ V6c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V7v1))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V8v2))))))))) \Rightarrow (\forall V9v \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V10v1 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V11v2 \in (ty\_2Eoption\_2Eoption\ A\_27b).(p\ (ap\ (ap\ (ap\ V0P\ V9v)\ V10v1)\ V11v2))))))) \tag{24}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0cmp \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V1v1 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2v2 \in (ty\_2Eoption\_2Eoption\ A\_27b).(\forall V3cmp\_27 \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). (\forall V4v1\_27 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V5v2\_27 \in (ty\_2Eoption\_2Eoption\ A\_27b).((((V1v1 = V4v1\_27) \land ((V2v2 = V5v2\_27) \land (\forall V6x \in A\_27a.(\forall V7x\_27 \in A\_27b.(((V4v1\_27 = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V6x)) \land (V5v2\_27 = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V7x\_27))) \Rightarrow ((ap\ (ap\ V0cmp\ V6x)\ V7x\_27) = (ap\ (ap\ V3cmp\_27\ V6x)\ V7x\_27))))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V0cmp)\ V1v1)\ V2v2) = (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V3cmp\_27)\ V4v1\_27)\ V5v2\_27)))))))))$$