thm_2Ecomparison_2Eoption__cmp__def (TMXFM-WoEsG5RcazfNfRi5wbGjxEHJ55mcWa)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{4}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{5}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (6)$$

Definition 6 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (7)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (8)$$

Definition 7 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (9)$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A{\wedge}p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 9 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 13 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2EternaryComparisons\_2Eoption\_\_compare : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2EternaryComparis\ A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Eoption\_2Eoption\ A\_27b)})^{(ty\_2Eoption\_2Eoption} \quad (10)$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). \\
(\forall V1v0 \in A\_27b.(\forall V2v3 \in A\_27a.(\forall V3v1 \in A\_27a. \\
(\forall V4v2 \in A\_27b.(((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare \\
A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27b)) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_ \\
A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27b)\ V1v0)) = c\_2EternaryComparisons\_2ELESS) \wedge (((ap\ (ap\ (ap \\
(c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V0c) \\
(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2v3))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)) = \\
c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_com \\
A\_27a\ A\_27b)\ V0c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3v1))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27b)\ V4v2)) = (ap\ (ap\ V0c\ V3v1)\ V4v2))))))))))
\end{gathered}
\tag{11}
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Theorem 1

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\
\forall V0c \in ((ty\_2EternaryComparisons\_2Eordering^{A\_27b})^{A\_27a}). \\
(\forall V1v0 \in A\_27b.(\forall V2v3 \in A\_27a.(\forall V3v1 \in A\_27a. \\
(\forall V4v2 \in A\_27b.(((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_compare \\
A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27b)) = c\_2EternaryComparisons\_2EEQUAL) \wedge (((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_ \\
A\_27a\ A\_27b)\ V0c)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27b)\ V1v0)) = c\_2EternaryComparisons\_2ELESS) \wedge (((ap\ (ap\ (ap \\
(c\_2EternaryComparisons\_2Eoption\_\_compare\ A\_27a\ A\_27b)\ V0c) \\
(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2v3))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)) = \\
c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Eoption\_\_com \\
A\_27a\ A\_27b)\ V0c)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3v1))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27b)\ V4v2)) = (ap\ (ap\ V0c\ V3v1)\ V4v2))))))))))
\end{gathered}
$$