

# thm\_2Ecomplex\_2ECOMPLEX\_\_DIV\_\_DENOM\_\_CANCEL (TMbZEveGeZrf2JRm159663H6T45mKmacFLo)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2ESUC\_REP\ m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 9** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (7)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

**Definition 10** We define  $c\_2Ecomplex\_2EIM$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (10)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (11)$$

**Definition 11** We define  $c\_2Ecomplex\_2ERE$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_2REP\_2CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_2REP\_2CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (13)$$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap\ P\ x)) \mathbf{then} (the (\lambda x.x \in A \wedge P\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Erealax\_2Ereal\_2REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40\ a))$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (14)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (15)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}}) \quad (16)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (17)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (18)$$

**Definition 17** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (19)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 19** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 20** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 21** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (20)$$

**Definition 22** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

**Definition 23** We define  $c\_2Ecomplex\_2Ecomplex\_inv$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal)\ V0z$

**Definition 24** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.\lambda V2z \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Ereal\_2Ereal\_sub\ V0x\ V1y)\ V2z)$

**Definition 25** We define  $c\_2Ecomplex\_2Ecomplex\_mul$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal)\ V0z$

**Definition 26** We define  $c\_2Ecomplex\_2Ecomplex\_div$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal)\ V0z$

**Definition 27** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))$

**Definition 28** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ V0t1\ V1t2))\ (\lambda V2t \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ V2t)))$

**Definition 29** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 30** We define  $c\_2Ecomplex\_2Ecomplex\_of\_real$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V0x))$

**Definition 31** We define  $c\_2Ecomplex\_2Ecomplex\_of\_num$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Ecomplex\_2Ecomplex\_of\_real\ V0n)$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \vee \neg(p\ V0t)))) \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod \ ty\_2Erealx\_2Ereal \ ty\_2Erealx\_2Ereal). \\
& (\forall V1w \in (ty\_2Epair\_2Eprod \ ty\_2Erealx\_2Ereal \ ty\_2Erealx\_2Ereal). \\
& ((ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \ V0z) \ V1w) = (ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \\
& \ V1w) \ V0z))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod \ ty\_2Erealx\_2Ereal \ ty\_2Erealx\_2Ereal). \\
& (\forall V1w \in (ty\_2Epair\_2Eprod \ ty\_2Erealx\_2Ereal \ ty\_2Erealx\_2Ereal). \\
& (\forall V2v \in (ty\_2Epair\_2Eprod \ ty\_2Erealx\_2Ereal \ ty\_2Erealx\_2Ereal). \\
& ((ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \ V0z) \ (ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \\
& \ V1w) \ V2v)) = (ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \ (ap \ (ap \ c\_2Ecomplex\_2Ecomplex\_mul \\
& \ V0z) \ V1w)) \ V2v))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) = \\
& \quad V0z))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& ((\neg(V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)))) \Rightarrow \\
& ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ (ap\ c\_2Ecomplex\_2Ecomplex\_inv \\
& V0z)) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num \\
& c\_2Enum\_2E0)) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num \\
& c\_2Enum\_2E0))\ V0z) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& (\forall V1w \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& (\forall V2v \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& (((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ V1w) = (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul \\
& V0z)\ V2v)) \Leftrightarrow ((V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)) \vee \\
& \quad (V1w = V2v))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& (\forall V1w \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\
& (((\neg(V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0))) \wedge \\
& \quad (\neg(V1w = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)))) \Rightarrow \\
& ((ap\ c\_2Ecomplex\_2Ecomplex\_inv\ (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul \\
& V0z)\ V1w)) = (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ (ap\ c\_2Ecomplex\_2Ecomplex\_inv \\
& \quad V0z))\ (ap\ c\_2Ecomplex\_2Ecomplex\_inv\ V1w))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)). \\
& ((\neg(V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0))) \Rightarrow \\
& ((ap\ c\_2Ecomplex\_2Ecomplex\_inv\ (ap\ c\_2Ecomplex\_2Ecomplex\_inv \\
& \quad V0z)) = V0z)))
\end{aligned} \tag{41}$$

Assume the following.

$$((ap\ c\_2Ecomplex\_2Ecomplex\_inv\ (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)) \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)). \\
& (((ap\ c\_2Ecomplex\_2Ecomplex\_inv\ V0z) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)) \Leftrightarrow (V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0)))
\end{aligned} \tag{43}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)). \\
& (\forall V1w \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)). \\
& (\forall V2v \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)). \\
& ((\neg(V0z = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_num\ c\_2Enum\_2E0))) \Rightarrow \\
& ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_div\ (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_div \\
& \quad V1w)\ V0z))\ (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_div\ V2v)\ V0z)) = (ap\ ( \\
& \quad ap\ c\_2Ecomplex\_2Ecomplex\_div\ V1w)\ V2v))))))
\end{aligned}$$