

thm_2Ecomplex_2Ecomplex__EQ__LMUL__IMP
(TMMKW-
peso2Av1rjjUu3WiDrwfVZpeXH8o2E)

October 26, 2020

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A.27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 4 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \tag{4}$$

Definition 5 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 6 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 7 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$. $(ap\ (c_2Emin_2E.40\ (ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (9)$$

Definition 8 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 9 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}} \quad (10)$$

Definition 10 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (11)$$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $(ap\ c_2Erealax_2Ereal$

Definition 12 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal$. $\lambda V1y \in ty_2Erealax_2Ereal$

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V0t))$
Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (12)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 16 We define $c_2Ecomplex_2Ecomplex_mul$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$
Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 17 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (16)$$

Definition 18 We define $c_2Ecomplex_2Ecomplex_of_real$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (c_2E$

Definition 19 We define $c_2Ecomplex_2Ecomplex_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Ecomp$

Definition 20 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$\begin{aligned} (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ (\forall V1w \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ ((ap (ap c_2Ecomplex_2Ecomplex_mul V0z) V1w) = (ap (ap c_2Ecomplex_2Ecomplex_mul \\ V1w) V0z)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (\forall V1w \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (\forall V2v \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (((\neg(V0z = (ap\ c_2Ecomplex_2Ecomplex_of_num\ c_2Enum_2E0))) \wedge \\
& ((ap\ (ap\ c_2Ecomplex_2Ecomplex_mul\ V1w)\ V0z) = (ap\ (ap\ c_2Ecomplex_2Ecomplex_mul \\
& \quad V2v)\ V0z))) \Rightarrow (V1w = V2v))))
\end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (\forall V1w \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (\forall V2v \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)). \\
& (((\neg(V0z = (ap\ c_2Ecomplex_2Ecomplex_of_num\ c_2Enum_2E0))) \wedge \\
& ((ap\ (ap\ c_2Ecomplex_2Ecomplex_mul\ V0z)\ V1w) = (ap\ (ap\ c_2Ecomplex_2Ecomplex_mul \\
& \quad V0z)\ V2v))) \Rightarrow (V1w = V2v))))
\end{aligned}$$