

thm_2Ecomplex_2ECOMPLEX__EXP__NEG__MUL2 (TMMCV2is15grLYGhuEbpcSzZvAX2iqFNM1R)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (7)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (8)$$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2)) (\lambda V2t \in 2)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b))^{((2^{A_27b})^{A_27a})} \quad (10)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod))$

Definition 12 We define $c_2Ecomplex_2Ecomplex_of_real$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (c_2Epair_2E_2C)))$

Definition 13 We define $c_2Ecomplex_2Ecomplex_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Ecomplex_2Ecomplex_of_real)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (11)$$

Definition 14 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (13)$$

Definition 15 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** $(\lambda x.x \in A \wedge \dots)$ of type $\iota \Rightarrow \iota$.

Definition 16 We define `c_2Erealax_2Ereal__REP` to be $\lambda V0a \in ty_2Erealax_2Ereal$. $(ap (c_2Emin_2E_40 (t$

Let `c_2Erealax_2Etreax__neg` : ι be given. Assume the following.

$$c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Let `c_2Erealax_2Etreax__eq` : ι be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Let `c_2Erealax_2Ereal__ABS__CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (16)$$

Definition 17 We define `c_2Erealax_2Ereal__ABS` to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 18 We define `c_2Erealax_2Ereal__neg` to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $(ap\ c_2Erealax_2Ereal$

Let `c_2Epair_2EFAST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFAST\ A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \quad (17)$$

Definition 19 We define `c_2Ecomplex_2ERE` to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)$

Definition 20 We define `c_2Ecomplex_2Ecomplex__neg` to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal$

Let `c_2Ereal_2Epow` : ι be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Eenum_2Eenum})_{ty_2Erealax_2Ereal} \quad (18)$$

Let `c_2Earithmetic_2EFACT` : ι be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum} \quad (19)$$

Definition 21 We define `c_2Earithmetic_2EBIT2` to be $\lambda V0n \in ty_2Eenum_2Eenum$. $(ap (ap\ c_2Earithmetic$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})_{ty_2Eenum_2Eenum} \quad (20)$$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})_{ty_2Eenum_2Eenum} \quad (21)$$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (22)$$

Definition 22 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (23)$$

Definition 23 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 24 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (24)$$

Definition 25 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (25)$$

Definition 27 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 28 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 29 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 30 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 31 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 32 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 33 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 34 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 43 We define $c_Eseq_Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 44 We define $c_Etransc_Esin$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 45 We define $c_Etransc_Ecos$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 46 We define $c_Etransc_Eexp$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 47 We define $c_Ecomplex_Ecomplex_scalar_mul$ to be $\lambda V0k \in ty_2Erealax_2Ereal.\lambda V1z \in$

Definition 48 We define $c_Ecomplex_Ecomplex_exp$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$

Definition 49 We define $c_Ecomplex_Ecomplex_mul$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & (\forall V1w \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & ((ap (ap c_2Ecomplex_2Ecomplex_mul V0z) V1w) = (ap (ap c_2Ecomplex_2Ecomplex_mul \\ & \quad V1w) V0z)))) \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & ((ap (ap c_2Ecomplex_2Ecomplex_mul (ap c_2Ecomplex_2Ecomplex_exp \\ & \quad V0z)) (ap c_2Ecomplex_2Ecomplex_exp (ap c_2Ecomplex_2Ecomplex_neg \\ & \quad V0z))) = (ap c_2Ecomplex_2Ecomplex_of_num (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & ((ap (ap c_2Ecomplex_2Ecomplex_mul (ap c_2Ecomplex_2Ecomplex_exp \\ & \quad (ap c_2Ecomplex_2Ecomplex_neg V0z))) (ap c_2Ecomplex_2Ecomplex_exp \\ & \quad V0z))) = (ap c_2Ecomplex_2Ecomplex_of_num (ap c_2Earithmetic_2ENUMERAL \\ & \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned}$$