

thm\_2Ecomplex\_2ECOMPLEX\_\_MUL\_\_RCONJ1  
 (TMEtaqnZCvPhBBppM-  
 BQLrJ61pm3C4rGDccf)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{4}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow Q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2)) (\lambda V2t \in 2.$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

**Definition 8** We define  $c\_2Ecomplex\_2Ecomplex\_of\_real$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap\ (ap\ (c\_2Epair\_2E\_2C\ x\ x)\ V0)\ V0)$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

**Definition 9** We define  $c\_2Ecomplex\_2EIM$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (9)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \quad (10)$$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ a)\ V0)$

Let  $c\_2Erealx\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

Let  $c\_2Erealx\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (12)$$

Let  $c\_2Erealx\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_ABS\_CLASS \in (ty\_2Erealx\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (13)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ereal)$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal)$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (14)$$

**Definition 14** We define  $c\_2Ecomplex\_2ERE$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)$

**Definition 15** We define  $c\_2Ecomplex\_2Econj$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (15)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}} \end{aligned} \quad (16)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 18** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 19** We define  $c\_2Ecomplex\_2Ecomplex\_mul$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal)$

**Definition 20** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (17)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (18)$$

**Definition 21** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (19)$$

**Definition 22** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B))$

**Definition 23** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Ereal_2Epow` :  $\iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (20)$$

Let `c_2Erealax_2Etreallt` :  $\iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (21)$$

**Definition 24** We define `c_2Erealax_2Ereal_lt` to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 25** We define `c_2Etransc_2Eroot` to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1x \in ty\_2Erealax\_2Ereal$ .

**Definition 26** We define `c_2Etransc_2Esqrt` to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ c\_2Etransc\_2Eroot\ ($

**Definition 27** We define `c_2Ecomplex_2Emodu` to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\ & ((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ (ap\ c\_2Ecomplex\_2Econj\ V0z)) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_real\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2ERE\ V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))))\ (ap\ (ap \\ & c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2EIM\ V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal). \\ & ((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2Emodu\ V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))) = (ap\ (ap \\ & c\_2Erealax\_2Ereal\_add\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2ERE \\ & V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2 \\ & c\_2Earithmetic\_2EZERO))))\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2EIM \\ & V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2 \\ & c\_2Earithmetic\_2EZERO)))))) \end{aligned} \quad (26)$$

**Theorem 1**

$(\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)).$   
 $((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_mul\ V0z)\ (ap\ c\_2Ecomplex\_2Econj\ V0z)) = (ap\ c\_2Ecomplex\_2Ecomplex\_of\_real\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2Emodu\ V0z))\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))))$