

# thm\_2Ecomplex\_2ECOMPLEX\_\_POW\_\_L (TM- cjGLsq2nKPB9SKgoQ2EzQxw6Nw1GyuDho)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ecomplex\_2EIM$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Erealax\_2Ereal}) \tag{5}$$

**Definition 5** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x)) then (the (\lambda x.x \in A \wedge p (ap\ P\ x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 6** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty\_2Erealax\_2Ereal\_mul) (c\_2Erealax\_2Ereal\_mul)))$ .  
Let  $c\_2Erealax\_2Ereal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)} \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal))_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)} \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)} \quad (8)$$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$ .

**Definition 8** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Epair\_2E\_EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2E\_EFST A\_27a A\_27b \in (A\_27a (ty\_2Epair\_2Eprod A\_27a A\_27b)) \quad (9)$$

**Definition 9** We define  $c\_2Ecomplex\_2ERE$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal)$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.t)))$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b} A\_27a)}) \quad (10)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2E\_21 2) (\lambda V2z \in 2.z)))$ .

**Definition 13** We define  $c\_2Ecomplex\_2Ecomplex\_scalar\_mul$  to be  $\lambda V0k \in ty\_2Erealax\_2Ereal.\lambda V1z \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Erealax\_2Ereal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)})_{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)} \quad (11)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (12)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_2Erealax\_2Ereal$

**Definition 16** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 17** We define  $c\_2Ecomplex\_2Ecomplex\_mul$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \quad (13)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Eenum\_2Eenum}) \quad (14)$$

Let  $c\_2Eenum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EZERO\_REP \in \omega \quad (15)$$

Let  $c\_2Eenum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EABS\_num \in (ty\_2Eenum\_2Eenum^{\omega}) \quad (16)$$

**Definition 18** We define  $c\_2Eenum\_2E0$  to be  $(ap\ c\_2Eenum\_2EABS\_num\ c\_2Eenum\_2EZERO\_REP)$ .

**Definition 19** We define  $c\_2Ecomplex\_2Ecomplex\_of\_real$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (c\_2E$

**Definition 20** We define  $c\_2Ecomplex\_2Ecomplex\_of\_num$  to be  $\lambda V0n \in ty\_2Eenum\_2Eenum. (ap\ c\_2Ecomp$

Let  $c\_2Ecomplex\_2Ecomplex\_pow : \iota$  be given. Assume the following.

$$c\_2Ecomplex\_2Ecomplex\_pow \in (((ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)^{ty\_2Eenum\_2Eenum}) (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)) \quad (17)$$

Let  $c\_2Eenum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EREP\_num \in (\omega^{ty\_2Eenum\_2Eenum}) \quad (18)$$

Let  $c\_2Eenum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2ESUC\_REP \in (\omega^{\omega}) \quad (19)$$

**Definition 21** We define  $c\_2Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum. (ap\ c\_2Eenum\_2EABS\_num$

**Definition 22** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Eenum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (20)$$

**Definition 23** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 24** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal} \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0k \in ty\_2Erealax\_2Ereal.(\forall V1l \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & (\forall V3w \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_mul (ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul \\ & V0k) V2z)) (ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul V1l) V3w)) = \\ & (ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul (ap (ap c\_2Erealax\_2Ereal\_mul \\ & V0k) V1l)) (ap (ap c\_2Ecomplex\_2Ecomplex\_mul V2z) V3w))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul (ap c\_2Ereal\_2Ereal\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\ & V0z) = V0z)) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_pow V0z) c\_2Enum\_2E0) = (ap c\_2Ecomplex\_2Ecomplex\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\ & (\forall V1z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Ecomplex\_2Ecomplex\_pow \\ & V1z) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Ecomplex\_2Ecomplex\_mul \\ & V1z) (ap (ap c\_2Ecomplex\_2Ecomplex\_pow V1z) V2n)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Ereal\_2Epow V0x) \\
& c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
& ty\_2Erealax\_2Ereal.(\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Ereal\_2Epow \\
& V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{29}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\
& ((ap (ap c\_2Ecomplex\_2Ecomplex\_pow (ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul \\
& V1k) V2z)) V0n) = (ap (ap c\_2Ecomplex\_2Ecomplex\_scalar\_lmul \\
& (ap (ap c\_2Ereal\_2Epow V1k) V0n)) (ap (ap c\_2Ecomplex\_2Ecomplex\_pow \\
& V2z) V0n))))))
\end{aligned}$$