

thm_2Ecomplex_2ECOMPLEX__POW__ONE
 (TMUrJm-
 mZjFDXFezbA7dei3ApWEPSHW4ViGU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{3}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (V0P))))$

Definition 4 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{4}$$

Definition 5 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (5)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (6)$$

Definition 6 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 7 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (7)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (8)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (9)$$

Definition 8 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 9 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (10)$$

Definition 10 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Ereal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 12 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 13 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2)))$
Let $c_Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (12)$$

Definition 15 We define $c_Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 16 We define $c_Ecomplex_2Ecomplex_mul$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2E$
Let $c_Eenum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_2EZERO_REP \in \omega \quad (13)$$

Let $ty_2Eenum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Enum \quad (14)$$

Let $c_Eenum_2EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_2EABS_num \in (ty_2Eenum_2Enum^{\omega}) \quad (15)$$

Definition 17 We define c_Eenum_2E0 to be $(ap c_Eenum_2EABS_num c_Eenum_2EZERO_REP)$.

Definition 18 We define $c_Earithmic_2EZERO$ to be c_Eenum_2E0 .

Let $c_Eenum_2EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_2EREP_num \in (\omega^{ty_2Eenum_2Enum}) \quad (16)$$

Let $c_Eenum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 19 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Enum.(ap c_Eenum_2EABS_num$

Let $c_Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_Earithmic_2E_2B \in ((ty_2Eenum_2Enum^{ty_2Eenum_2Enum})^{ty_2Eenum_2Enum}) \quad (18)$$

Definition 20 We define $c_Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Eenum_2Enum.(ap (ap c_Earithmic$

Definition 21 We define $c_Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Eenum_2Enum.V0x$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (19)$$

Definition 22 We define $c_2Ecomplex_2Ecomplex_of_real$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (c_2Ereal_2Ereal_of_num$

Definition 23 We define $c_2Ecomplex_2Ecomplex_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Ecomplex_2Ecomplex_of_num$

Let $c_2Ecomplex_2Ecomplex_pow : \iota$ be given. Assume the following.

$$c_2Ecomplex_2Ecomplex_pow \in (((ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal) ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{(ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)} \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & ((ap (ap c_2Ecomplex_2Ecomplex_mul (ap c_2Ecomplex_2Ecomplex_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\ & V0z) = V0z)) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & ((\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & ((ap (ap c_2Ecomplex_2Ecomplex_pow V0z) c_2Enum_2E0) = (ap c_2Ecomplex_2Ecomplex_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\ & (\forall V1z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\ & (\forall V2n \in ty_2Enum_2Enum. ((ap (ap c_2Ecomplex_2Ecomplex_pow \\ & V1z) (ap c_2Enum_2ESUC V2n)) = (ap (ap c_2Ecomplex_2Ecomplex_mul \\ & V1z) (ap (ap c_2Ecomplex_2Ecomplex_pow V1z) V2n)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}). ((p (ap V0P c_2Enum_2E0)) \wedge \\ & (\forall V1n \in ty_2Enum_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap V0P V2n)))))) \end{aligned} \quad (25)$$

Theorem 1

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Ecomplex_2Ecomplex_pow \\ & (ap c_2Ecomplex_2Ecomplex_of_num (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0n) = (\\ & ap c_2Ecomplex_2Ecomplex_of_num (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned}$$