thm_2Ecomplex_2ECOMPLEX__RSCALAR__RMUL__SUB (TMF-GaFmf2XR6MKURDt1Dh4zHidbboppV5A7)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ecomplex_2EIM *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Er$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{4}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{5}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{6}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{8}$$

Definition 7 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 8 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{9}$$

Definition 9 *We define* c_2Ecomplex_2ERE *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Er$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 13 *We define* c_2Ecomplex_2Ecomplex__neg *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{11}$$

Definition 14 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Definition 15 *We define* c_2Ecomplex_2Ecomplex__add *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Er$

Definition 16 *We define* c_2Ecomplex_2Ecomplex__sub *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Er$

Definition 17 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{12}$$

Definition 18 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Definition 19 *We define* c_2Ecomplex_2Ecomplex__scalar__rmul *to be* $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erea$

Definition 20 *We define* c_2Ecomplex_2Ecomplex__scalar__lmul *to be* $\lambda V0k \in ty\_2Erealax\_2Ereal.\lambda V1z \in$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0k \in ty\_2Erealax\_2Ereal.(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal).((ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_scalar\_\_lmul}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0k)\ V1l))\ V2z) = (ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_scalar\_\_lmul}\ V0k)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_scalar\_\_lmul}\ V1l)\ V2z)))))) \tag{17}$$

Assume the following.

$$(\forall V0k \in ty\_2Erealax\_2Ereal.(\forall V1z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal).((ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_scalar\_\_lmul}\ V0k)\ V1z) = (ap\ (ap\ \mathsf{c\_2Ecomplex\_2Ecomplex\_\_scalar\_\_rmul}\ V1z)\ V0k)))) \tag{18}$$

Theorem 1

$$(\forall V0k \in ty_2Erealax_2Ereal.(\forall V1l \in ty_2Erealax_2Ereal.
(\forall V2z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal).
((ap\ (ap\ \mathsf{c_2Ecomplex_2Ecomplex__scalar__rmul}\ V2z)\ (ap\ (ap\ \mathsf{c_2Ereal_2Ereal__sub}\ V0k)\ V1l)) = (ap\ (ap\ \mathsf{c_2Ecomplex_2Ecomplex__sub}\ (ap\ (ap\ \mathsf{c_2Ecomplex_2Ecomplex__scalar__rmul}\ V2z)\ V0k))\ (ap\ (ap\ \mathsf{c_2Ecomplex_2Ecomplex__scalar__rmul}\ V2z)\ V1l))))))$$