

thm_2Ecomplex_2ECOMPLEX_SCALAR_LMUL_ADD (TMHaQJmyM3J9GAodBEixXJsuXZSUXdb9t6U)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{3}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{4}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Erealax_2Ereal_REP_CLASS}) \tag{5}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E.40 (ty_2Erealax_2Ereal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (6)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (7)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (8)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFAST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (9)$$

Definition 9 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)$

Definition 10 We define $c_2Emin_2E.3D.3D.3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E.2F.5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E.21 2) (\lambda V2t \in 2.))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b} A_27a)}) \quad (10)$$

Definition 12 We define $c_2Epair_2E.2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2E.2C$

Definition 13 We define $c_2Ecomplex_2Ecomplex_add$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)$

Let $c_2Erealax_2Ereal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (11)$$

Definition 14 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 15 We define $c_Ecomplex_Ecomplex_scalar_Imul$ to be $\lambda V0k \in ty_2Erealax_2Ereal.\lambda V1z \in$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul \\ & (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))\ V2z) = (ap\ (ap\ c_2Erealax_2Ereal_add \\ & (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V2z))\ (ap\ (ap\ c_2Erealax_2Ereal_mul \\ & V1y)\ V2z)))))) \end{aligned} \quad (18)$$

Theorem 1

$$\begin{aligned} & (\forall V0k \in ty_2Erealax_2Ereal.(\forall V1l \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal). \\ & ((ap\ (ap\ c_2Ecomplex_2Ecomplex_scalar_Imul\ (ap\ (ap\ c_2Erealax_2Ereal_add \\ & V0k)\ V1l))\ V2z) = (ap\ (ap\ c_2Ecomplex_2Ecomplex_add\ (ap\ (ap\ c_2Ecomplex_2Ecomplex_scalar_Imul \\ & V0k)\ V2z))\ (ap\ (ap\ c_2Ecomplex_2Ecomplex_scalar_Imul\ V1l)\ V2z)))))) \end{aligned}$$