

thm_2Ecomplex_2ECOMPLEX__SCALAR__LMUL__ENTIRE
(TMLsH-
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October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 8 We define $c_2Ecomplex_2Ecomplex_of_real$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (c_2E$

Definition 9 We define $c_2Ecomplex_2Ecomplex_of_num$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Ecompl$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (8)$$

Definition 10 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2E$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (9)$$

Definition 11 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2E$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (10)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (11)$$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x) \text{ of type } \iota \Rightarrow \iota).$

Definition 13 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (t$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (12)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (13)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (14)$$

Definition 14 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 15 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ecomplex_2Ecomplex_scalar_lmul$ to be $\lambda V0k \in ty_2Erealax_2Ereal.\lambda V1z \in ty_2Erealax_2Ereal$

Definition 17 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (15)$$

Definition 18 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 19 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 20 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Definition 21 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E2B\ n))$

Definition 22 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (19)$$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \wedge (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal). \\ & ((V0z = (ap\ c_2Ecomplex_2Ecomplex_of_num\ c_2Enum_2E0)) \Leftrightarrow ((\\ & ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ecomplex_2ERE \\ & V0z))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\ & c_2Earithmetic_2EZERO))))\ (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ecomplex_2EIM \\ & V0z))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\ & c_2Earithmetic_2EZERO)))) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2ESND\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) \Leftrightarrow ((V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \vee \\
& (V1y = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V0x)) (ap (ap c_2Erealax_2Ereal_mul V1y) V1y)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) \Leftrightarrow ((V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \wedge \\
& (V1y = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Epow V0x) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) = \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V0x)))
\end{aligned} \tag{30}$$

Theorem 1

$$\begin{aligned}
& (\forall V0k \in ty_2Erealax_2Ereal. (\forall V1z \in (ty_2Epair_2Eprod \\
& ty_2Erealax_2Ereal ty_2Erealax_2Ereal). (((ap (ap c_2Ecomplex_2Ecomplex_scalar_lmul \\
& V0k) V1z) = (ap c_2Ecomplex_2Ecomplex_of_num c_2Enum_2E0)) \Leftrightarrow \\
& ((V0k = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \vee (V1z = (ap \\
& c_2Ecomplex_2Ecomplex_of_num c_2Enum_2E0))))))
\end{aligned}$$