

thm_2Ecomplex_2ECOMPLEX__SUB__ADD (TMJ9g2NVnwweG6ZT71WJ7QwuYWikEy3WcjV)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \tag{3}$$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a}))) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 4 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{4}$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \tag{5}$$

Definition 5 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x)) then (the (\lambda x.x \in A \wedge p (ap\ P\ x)))$ of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (7)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})) \quad (8)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_neg$

Let $c_2Epair_2E_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2E_EFST A_27a A_27b \in (A_27a (ty_2Epair_2Eprod A_27a A_27b)) \quad (9)$$

Definition 9 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.t1 t2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2E_2C$

Definition 13 We define $c_2Ecomplex_2Ecomplex_neg$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (11)$$

Definition 14 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 15 We define $c_Ecomplex_Ecomplex_add$ to be $\lambda V0z \in (ty_Epair_Eprod\ ty_Erealax_Ereal)$

Definition 16 We define $c_Ecomplex_Ecomplex_sub$ to be $\lambda V0z \in (ty_Epair_Eprod\ ty_Erealax_Ereal)$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \tag{12}$$

Let $ty_Eenum_Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_Eenum_Eenum \tag{13}$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum^{\omega}) \tag{14}$$

Definition 17 We define c_Eenum_E0 to be $(ap\ c_Eenum_EABS_num\ c_Eenum_EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \tag{15}$$

Definition 18 We define $c_Ecomplex_Ecomplex_of_real$ to be $\lambda V0x \in ty_Erealax_Ereal.(ap\ (ap\ (c_Eenum_E0\ V0x)))$

Definition 19 We define $c_Ecomplex_Ecomplex_of_num$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ c_Ecomplex_Ecomplex_of_real\ V0n)$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{17}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{19}$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_Epair_Eprod\ ty_Erealax_Ereal\ ty_Erealax_Ereal). \\ & (\forall V1w \in (ty_Epair_Eprod\ ty_Erealax_Ereal\ ty_Erealax_Ereal). \\ & (\forall V2v \in (ty_Epair_Eprod\ ty_Erealax_Ereal\ ty_Erealax_Ereal). \\ & ((ap\ (ap\ c_Ecomplex_Ecomplex_add\ V0z)\ (ap\ (ap\ c_Ecomplex_Ecomplex_add\ V1w)\ V2v)) = (ap\ (ap\ c_Ecomplex_Ecomplex_add\ (ap\ (ap\ c_Ecomplex_Ecomplex_add\ V0z)\ V1w))\ V2v)))))) \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)). \\
& ((ap\ (ap\ c_2Ecomplex_2Ecomplex_add\ V0z)\ (ap\ c_2Ecomplex_2Ecomplex_of_num \\
& \quad c_2Enum_2E0)) = V0z))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)). \\
& ((ap\ (ap\ c_2Ecomplex_2Ecomplex_add\ (ap\ c_2Ecomplex_2Ecomplex_neg \\
& \quad V0z))\ V0z) = (ap\ c_2Ecomplex_2Ecomplex_of_num\ c_2Enum_2E0)))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)). \\
& (\forall V1w \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)). \\
& ((ap\ (ap\ c_2Ecomplex_2Ecomplex_add\ (ap\ (ap\ c_2Ecomplex_2Ecomplex_sub \\
& \quad V0z)\ V1w))\ V1w) = V0z)))
\end{aligned}$$