

thm_2Ecomplex_2ECOMPLEX__SUB__SCALAR__RMUL
(TMG-
WjXMb4TL7j7qwN7q5aVpHrAPYNYtKV4j)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \tag{3}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 4 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{4}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{5}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal$.(ap $(c_2Emin_2E_40$ (ty

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (7)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (8)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$.(ap $c_2Erealax_2Ereal$

Let $c_2Epair_2EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFAST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (9)$$

Definition 9 We define $c_2Ecomplex_2ERE$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Ehreal_2Ehreal)$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b} A_27a})}) \quad (10)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b$.(ap $(c_2Epair_2EABS_prod$

Definition 13 We define $c_2Ecomplex_2Ecomplex_neg$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Ehreal_2Ehreal)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (11)$$

Definition 14 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 15 We define $c_2Ecomplex_2Ecomplex_add$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$

Definition 16 We define $c_2Ecomplex_2Ecomplex_sub$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}))^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)} \quad (12)$$

Definition 17 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 18 We define $c_2Ecomplex_2Ecomplex_scalar_rmul$ to be $\lambda V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal$

Definition 19 We define $c_2Ecomplex_2Ecomplex_scalar_lmul$ to be $\lambda V0k \in ty_2Erealax_2Ereal.\lambda V1z \in ty_2Erealax_2Ereal$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0k \in ty_2Erealax_2Ereal.(\forall V1z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal).(\forall V2w \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal).((ap (ap c_2Ecomplex_2Ecomplex_scalar_lmul V0k) (ap (ap c_2Ecomplex_2Ecomplex_sub V1z) V2w)) = (ap (ap c_2Ecomplex_2Ecomplex_sub (ap (ap c_2Ecomplex_2Ecomplex_scalar_lmul V0k) V1z)) (ap (ap c_2Ecomplex_2Ecomplex_scalar_lmul V0k) V2w))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0k \in ty_2Erealax_2Ereal.(\forall V1z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal).((ap (ap c_2Ecomplex_2Ecomplex_scalar_lmul V0k) V1z) = (ap (ap c_2Ecomplex_2Ecomplex_scalar_rmul V1z) V0k)))) \end{aligned} \quad (18)$$

Theorem 1

$$\begin{aligned} & (\forall V0k \in \text{ty_2Erealax_2Ereal}. (\forall V1z \in (\text{ty_2Epair_2Eprod} \\ & \text{ty_2Erealax_2Ereal ty_2Erealax_2Ereal}). (\forall V2w \in (\text{ty_2Epair_2Eprod} \\ & \text{ty_2Erealax_2Ereal ty_2Erealax_2Ereal}). ((\text{ap} (\text{ap} \text{c_2Ecomplex_2Ecomplex_scalar_rmul} \\ & (\text{ap} (\text{ap} \text{c_2Ecomplex_2Ecomplex_sub} V1z) V2w)) V0k) = (\text{ap} (\text{ap} \text{c_2Ecomplex_2Ecomplex_sub} \\ & (\text{ap} (\text{ap} \text{c_2Ecomplex_2Ecomplex_scalar_rmul} V1z) V0k)) (\text{ap} (\text{ap} \\ & \text{c_2Ecomplex_2Ecomplex_scalar_rmul} V2w) V0k)))))) \end{aligned}$$