

# thm\_2Ecomplex\_2EDE\_\_MOIVRE\_\_LEMMA (TMJmtfvwisdqkS2wRZ8ACVAYpacagZDJz5X)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 3** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{4}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

**Definition 4** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 7** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

**Definition 8** We define  $c\_2Ecomplex\_2Ecomplex\_of\_real$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (c\_2Epair\_2E\_2C\ x\ x)\ y)\ z)$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (8)$$

**Definition 9** We define  $c\_2Ecomplex\_2ERE$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

**Definition 10** We define  $c\_2Ecomplex\_2EIM$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (10)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (11)$$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ a)\ ty\_2Erealax\_2Ereal\_REP\_CLASS)$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Erealax\_2Ereal\_REP\_CLASS})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal} \quad (12)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})$$
(13)

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}$$
(14)

**Definition 13** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
(15)

**Definition 15** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})$$
(16)

**Definition 16** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

**Definition 17** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 18** We define  $c\_2Ecomplex\_2Ecomplex\_mul$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal$

**Definition 19** We define  $c\_2Ecomplex\_2Ecomplex\_of\_num$  to be  $\lambda V0n \in ty\_2Eenum\_2Eenum.(ap\ c\_2Ecomplex$

Let  $c\_2Ecomplex\_2Ecomplex\_pow : \iota$  be given. Assume the following.

$$c\_2Ecomplex\_2Ecomplex\_pow \in (((ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)^{(ty\_2Eenum\_2Eenum)})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)})$$
(17)

Let  $c\_2Eenum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EREP\_num \in (\omega^{ty\_2Eenum\_2Eenum})$$
(18)

Let  $c\_2Eenum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2ESUC\_REP \in (\omega^{\omega})$$
(19)

**Definition 20** We define  $c\_2Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.(ap\ c\_2Eenum\_2EABS\_num$

Let  $c\_Ereal\_Epow : \iota$  be given. Assume the following.

$$c\_Ereal\_Epow \in ((ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum})^{ty\_Erealax\_Ereal}) \quad (20)$$

Let  $c\_Earithmetic\_EFACT : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EFACT \in (ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum}) \quad (21)$$

**Definition 21** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Eenum\_E0$ .

Let  $c\_Earithmetic\_E2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E2B \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (22)$$

**Definition 22** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap (ap c\_Earithmetic\_EBIT2))$

**Definition 23** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.V0x$ .

**Definition 24** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap (ap c\_Earithmetic\_EBIT1))$

Let  $c\_Earithmetic\_E2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E2D \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (23)$$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (24)$$

Let  $c\_Erealax\_Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_inv \in ((ty\_Epair\_Eprod ty\_Ehreal\_Ehreal)^{ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal}) \quad (25)$$

**Definition 25** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap c\_Erealax\_Ereal\_ABS)$

**Definition 26** We define  $c\_Ereal\_E2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.(ap c\_Erealax\_Ereal\_E2F)$

Let  $c\_Earithmetic\_EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEVEN \in (2^{ty\_Eenum\_Eenum}) \quad (26)$$

**Definition 27** We define  $c\_Ebool\_E2EF$  to be  $(ap (c\_Ebool\_E2E21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 28** We define  $c\_Ebool\_E2COND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(ap c\_Ebool\_E2COND))))$

Let  $c\_Ereal\_Esum : \iota$  be given. Assume the following.

$$c\_Ereal\_Esum \in ((ty\_Erealax\_Ereal^{(ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum})})^{ty\_Epair\_Eprod ty\_Eenum\_Eenum}) \quad (27)$$

**Definition 29** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_7E))$

**Definition 30** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_2E\_40$

**Definition 31** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 33** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21 2) (\lambda V2t \in$

**Definition 34** We define  $c\_Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Erealax\_2Ereal\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_lt \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal)}) \quad (28)$$

**Definition 35** We define  $c\_Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 36** We define  $c\_Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 37** We define  $c\_Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_Ebool\_2ECONJ$

**Definition 38** We define  $c\_Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a}$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (29)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \quad (30)$$

**Definition 39** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c\_2Emetric\_2Emetric$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \quad (31)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (32)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^{A\_27a})})}) \quad (33)$$

**Definition 40** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a).(ap$   
 Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends \\ & A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A\_27b})^{A\_27b}))})_{A\_27a})_{(A\_27a)^{A\_27b}}) \end{aligned} \quad (34)$$

**Definition 41** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 42** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 43** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap (c\_2E$

**Definition 44** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap c\_2Eseq\_2Esuminf (\lambda V1n$

**Definition 45** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap c\_2Eseq\_2Esuminf (\lambda V1n$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_mul (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) (ap c\_2Etransc\_2Ecos V0x)) (ap c\_2Etransc\_2Esin \\ & V0x))) (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & (ap c\_2Etransc\_2Ecos V1y)) (ap c\_2Etransc\_2Esin V1y))) = (ap (ap \\ & (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) (ap \\ & c\_2Etransc\_2Ecos (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y))) ( \\ & ap c\_2Etransc\_2Esin (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & ((\forall V0z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_pow V0z) c\_2Enum\_2E0) = (ap c\_2Ecomplex\_2Ecomplex\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\ & (\forall V1z \in (ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal). \\ & (\forall V2n \in ty\_2Enum\_2Enum.((ap (ap c\_2Ecomplex\_2Ecomplex\_pow \\ & V1z) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Ecomplex\_2Ecomplex\_mul \\ & V1z) (ap (ap c\_2Ecomplex\_2Ecomplex\_pow V1z) V2n)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& V1y) V0x))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum.((ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Enum\_2ESUC V0n)) = (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal.((ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1y) V2z))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etrasc\_2Esin (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etrasc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{46}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Ecomplex\_2Ecomplex\_pow (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) (ap c\_2Etransc\_2Ecos V0x)) (ap c\_2Etransc\_2Esin \\ & V0x))) V1n) = (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) \\ & (ap c\_2Etransc\_2Ecos (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\ & V1n)) V0x))) (ap c\_2Etransc\_2Esin (ap (ap c\_2Erealax\_2Ereal\_mul \\ & (ap c\_2Ereal\_2Ereal\_of\_num V1n)) V0x)))))) \end{aligned}$$