

# thm\_2Ecomplex\_2EDE\_\_MOIVRE\_\_THM (TM- RxU9oxQGDMawPodVfBTy6ZU4cwJ5EAxKs)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{4}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \tag{5}$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealx\_2Ereal}) \tag{6}$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(\lambda p (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap (c\_2Enum\_2EABS\_num ($

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (10)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap (c\_2Earithmetic\_2E\_2B$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (11)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap (c\_2Earithmetic\_2E\_2B$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (12)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (13)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal\_REP\_CLASS}) \quad (14)$$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap (c\_2Emin\_2E\_40 (t$



Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (21)$$

**Definition 21** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$   
Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (22)$$

**Definition 22** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

**Definition 23** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 24** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 26** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 27** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (23)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (24)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECON$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (25)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (26)$$

**Definition 33** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c)^{A\_27a})$ .  
Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (27)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))}) \quad (28)$$

**Definition 34** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal))\ (ap\ (c\_2Emetric\_2Edist))$ .  
Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))^{(c\_2Emetric\_2Emr1\ A\_27a)}) \quad (29)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (30)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (31)$$

**Definition 35** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a). (ap\ (c\_2Emetric\_2Emtop))$ .  
Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ (2^{(A\_27b)^{A\_27b}})})^{A\_27a})^{(A\_27a)^{A\_27b}})) \quad (32)$$

**Definition 36** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal\ ty\_2Enum\_2Enum). \lambda V1x \in ty\_2Enum\_2Enum$ .

**Definition 37** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal\ ty\_2Enum\_2Enum). \lambda V1s \in ty\_2Enum\_2Enum$ .

**Definition 38** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal\ ty\_2Enum\_2Enum). (ap\ (c\_2Eseq\_2Esuminf))$ .

**Definition 39** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1n \in ty\_2Enum\_2Enum. x^n))$ .

**Definition 40** We define  $c\_2Etransc\_2Epi$  to be  $(ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_mul)))$ .

**Definition 41** We define  $c\_2Ecomplex\_2EIM$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Enum\_2Enum)$ .

**Definition 42** We define  $c\_2Ecomplex\_2ERE$  to be  $\lambda V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Enum\_2Enum)$ .

**Definition 43** We define  $c\_2Etransc\_2Eroot$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. \lambda V1x \in ty\_2Erealax\_2Ereal$ .

**Definition 44** We define  $c\_Etransc\_Esqrt$  to be  $\lambda V0x \in ty\_Erealax\_EReal.(ap (ap c\_Etransc\_ERoot ($

**Definition 45** We define  $c\_Ecomplex\_Emodu$  to be  $\lambda V0z \in (ty\_Epair\_Eprod ty\_Erealax\_EReal ty\_E$

**Definition 46** We define  $c\_Etransc\_Eacs$  to be  $\lambda V0y \in ty\_Erealax\_EReal.(ap (c\_Emin\_E40 ty\_Ere$

**Definition 47** We define  $c\_Ecomplex\_Earg$  to be  $\lambda V0z \in (ty\_Epair\_Eprod ty\_Erealax\_EReal ty\_E$

**Definition 48** We define  $c\_Ecomplex\_Ecomplex\_scalar\_lmul$  to be  $\lambda V0k \in ty\_Erealax\_EReal.\lambda V1z \in$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (33)$$

**Definition 49** We define  $c\_Etransc\_Esin$  to be  $\lambda V0x \in ty\_Erealax\_EReal.(ap c\_Eseq\_Esuminf (\lambda V1n$

Let  $c\_Ecomplex\_Ecomplex\_pow : \iota$  be given. Assume the following.

$$c\_Ecomplex\_Ecomplex\_pow \in (((ty\_Epair\_Eprod ty\_Erealax\_EReal ty\_Erealax\_EReal)^{ty\_Eenum\_Eenum})^{(ty\_Epair\_Eprod ty\_Erealax\_EReal ty\_Erealax\_EReal)}) \quad (34)$$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_Eenum\_Eenum.(\forall V1k \in ty\_Erealax\_EReal. \\ & (\forall V2z \in (ty\_Epair\_Eprod ty\_Erealax\_EReal ty\_Erealax\_EReal). \\ & ((ap (ap c\_Ecomplex\_Ecomplex\_pow (ap (ap c\_Ecomplex\_Ecomplex\_scalar\_lmul \\ & V1k) V2z)) V0n) = (ap (ap c\_Ecomplex\_Ecomplex\_scalar\_lmul \\ & (ap (ap c\_Ereal\_Epow V1k) V0n)) (ap (ap c\_Ecomplex\_Ecomplex\_pow \\ & V2z) V0n)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_Erealax\_EReal.(\forall V1n \in ty\_Eenum\_Eenum. \\ & ((ap (ap c\_Ecomplex\_Ecomplex\_pow (ap (ap (c\_Epair\_E2C ty\_Erealax\_EReal \\ & ty\_Erealax\_EReal) (ap c\_Etransc\_Ecos V0x)) (ap c\_Etransc\_Esin \\ & V0x))) V1n) = (ap (ap (c\_Epair\_E2C ty\_Erealax\_EReal ty\_Erealax\_EReal) \\ & (ap c\_Etransc\_Ecos (ap (ap c\_Erealax\_EReal\_mul (ap c\_Ereal\_Ereal\_of\_num \\ & V1n)) V0x))) (ap c\_Etransc\_Esin (ap (ap c\_Erealax\_EReal\_mul \\ & (ap c\_Ereal\_Ereal\_of\_num V1n)) V0x)))))) \end{aligned} \quad (39)$$

**Theorem 1**

$(\forall V0z \in (ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)).$   
 $(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_pow$   
 $(ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_scalar\_lmul\ (ap\ c\_2Ecomplex\_2Emodu$   
 $V0z))\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)$   
 $(ap\ c\_2Etransc\_2Ecos\ (ap\ c\_2Ecomplex\_2Earg\ V0z)))\ (ap\ c\_2Etransc\_2Esin$   
 $(ap\ c\_2Ecomplex\_2Earg\ V0z))))))\ V1n) = (ap\ (ap\ c\_2Ecomplex\_2Ecomplex\_scalar\_lmul$   
 $(ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ecomplex\_2Emodu\ V0z))\ V1n))\ (ap\ ($   
 $ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ ($   
 $ap\ c\_2Etransc\_2Ecos\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num$   
 $V1n))\ (ap\ c\_2Ecomplex\_2Earg\ V0z))))\ (ap\ c\_2Etransc\_2Esin\ (ap\ ($   
 $ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1n))$   
 $(ap\ c\_2Ecomplex\_2Earg\ V0z))))))$