

thm_2Ecomplex_2EEXP__IMAGINARY (TMH3AJ0qP2wzr56MwfGNzn4Nhx9zecKS1AY)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E3D (2^{A-27a}))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1) n)$

Definition 8 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (7)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (8)$$

Definition 9 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21) 2) (\lambda V2t \in 2.t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 11 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod) x y)$

Definition 12 We define $c_2Ecomplex_2Ei$ to be $(ap (ap (c_2Epair_2E2C\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal) ty_2Erealx_2Ereal))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (11)$$

Definition 13 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealx_2Ereal\ ty_2Erealx_2Ereal)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (12)$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \quad (13)$$

Definition 14 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 15 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap (c_Emin_E40 (t$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)))(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) \quad (14)$$

Let $c_Erealax_Etrealeq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealeq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (15)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})} \quad (16)$$

Definition 16 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)$

Definition 17 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal$

Let $c_Epair_EFAST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_Epair_EFAST A.27a A.27b \in (A.27a^{(ty_Epair_Eprod A.27a A.27b)}) \quad (17)$$

Definition 18 We define $c_Ecomplex_ERE$ to be $\lambda V0z \in (ty_Epair_Eprod ty_Erealax_Ereal ty_Erealax_Ereal)$

Definition 19 We define $c_Ecomplex_Ecomplex_scalar_mul$ to be $\lambda V0z \in (ty_Epair_Eprod ty_Erealax_Ereal ty_Erealax_Ereal)$

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealax_Ereal^{ty_Eenum_Eenum})^{ty_Erealax_Ereal}) \quad (18)$$

Let $c_Earithmetic_EFACT : \iota$ be given. Assume the following.

$$c_Earithmetic_EFACT \in (ty_Eenum_Eenum^{ty_Eenum_Eenum}) \quad (19)$$

Definition 20 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap (ap c_Earithmetic_EBIT2$

Let $c_Earithmetic_E2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E2D \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (20)$$

Let $c_Earithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmetic_EDIV \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (21)$$

Let $c_Erealax_Etrealmul_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul_neg \in ((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (22)$$

Definition 21 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (23)$$

Definition 22 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Definition 23 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (24)$$

Definition 24 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (25)$$

Definition 26 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Definition 27 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 28 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 29 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 30 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 31 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (26)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 33 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (27)$$

Definition 34 We define $c_2Erealx_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealx_2Ereal.\lambda V1T2 \in ty_2Erealx_2Ereal$.

Definition 35 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealx_2Ereal.\lambda V1y \in ty_2Erealx_2Ereal$.

Definition 36 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap (ap (ap (c_2Ebool_2ECONJ$

Definition 37 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (28)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric A_27a)^{(ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (29)$$

Definition 38 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealx_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Edist A_27a \in ((ty_2Erealx_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)}) \quad (30)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Etopology_2Etopology A0) \quad (31)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Etopology_2Etopology A_27a \in ((ty_2Etopology_2Etopology A_27a)^{(2^{(2^{A_27a})})}) \quad (32)$$

Definition 39 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Eextends A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (33)$$

Definition 40 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealx_2Ereal^{ty_2Eenum_2Eenum}).\lambda V1x$

Definition 41 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Eenum_2Eenum}).\lambda V1s \in ty_2E$

Definition 42 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealx_2Ereal^{ty_2Eenum_2Eenum}).(ap (c_2E$

Definition 43 We define $c_2Etransc_2Esin$ to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 44 We define $c_2Etransc_2Ecos$ to be $\lambda V0x \in ty_2Erealx_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 45 We define `c_2Ecomplex_2Ecomplex__scalar__lmul` to be $\lambda V0k \in ty_2Erealax_2Ereal.\lambda V1z \in$

Definition 46 We define `c_2Etransc_2Eexp` to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Eseq_2Esuminf\ (\lambda V1t \in$

Definition 47 We define `c_2Ecomplex_2Ecomplex__exp` to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal$

Assume the following.

$$True \tag{34}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{35}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal). \\ & ((ap\ (ap\ c_2Ecomplex_2Ecomplex_scalar_lmul\ (ap\ c_2Ereal_2Ereal_of_num \\ & (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\ & V0z) = V0z)) \end{aligned} \tag{37}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \tag{38}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \tag{39}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ V0x) = V0x)) \tag{40}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_mul\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)\ V0x) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \tag{41}$$

Assume the following.

$$\begin{aligned}
 & ((ap\ c_2Etransc_2Eexp\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = \\
 & \quad (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (\\
 & \quad \quad ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\
 & \hspace{15em} (42)
 \end{aligned}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ c_2Ecomplex_2Ecomplex_exp \\
 & \quad (ap\ (ap\ c_2Ecomplex_2Ecomplex_scalar_rmul\ c_2Ecomplex_2Ei) \\
 V0x)) = (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealax_2Ereal\ ty_2Erealax_2Ereal) \\
 & \quad (ap\ c_2Etransc_2Ecos\ V0x))\ (ap\ c_2Etransc_2Esin\ V0x))))
 \end{aligned}$$