

thm_2Ecomplex_2EMODU__POS
(TMZ2Hvm1Dq5d3Nm2ejWy2Dfxxc6FhyGkKaQ)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \tag{3}$$

Definition 7 We define $c_2Ecomplex_2EIM$ to be $\lambda V0z \in (ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \tag{4}$$

Definition 8 We define $c_Ecomplex_ERE$ to be $\lambda V0z \in (ty_Epair_Eprod\ ty_Erealx_Ereal\ ty_Ereal)$.
Let $c_EEnum_EZERO_REP : \iota$ be given. Assume the following.

$$c_EEnum_EZERO_REP \in \omega \tag{5}$$

Let $ty_EEnum_EEnum : \iota$ be given. Assume the following.

$$nonempty\ ty_EEnum_EEnum \tag{6}$$

Let $c_EEnum_EABS_num : \iota$ be given. Assume the following.

$$c_EEnum_EABS_num \in (ty_EEnum_EEnum^{\omega}) \tag{7}$$

Definition 9 We define c_EEnum_E0 to be $(ap\ c_EEnum_EABS_num\ c_EEnum_EZERO_REP)$.

Definition 10 We define $c_Earithmic_EZERO$ to be c_EEnum_E0 .

Let $c_EEnum_EREP_num : \iota$ be given. Assume the following.

$$c_EEnum_EREP_num \in (\omega^{ty_EEnum_EEnum}) \tag{8}$$

Let $c_EEnum_ESUC_REP : \iota$ be given. Assume the following.

$$c_EEnum_ESUC_REP \in (\omega^{\omega}) \tag{9}$$

Definition 11 We define c_EEnum_ESUC to be $\lambda V0m \in ty_EEnum_EEnum.(ap\ c_EEnum_EABS_num\ m)$

Let $c_Earithmic_E2B : \iota$ be given. Assume the following.

$$c_Earithmic_E2B \in ((ty_EEnum_EEnum^{ty_EEnum_EEnum})^{ty_EEnum_EEnum}) \tag{10}$$

Definition 12 We define $c_Earithmic_EBIT2$ to be $\lambda V0n \in ty_EEnum_EEnum.(ap\ (ap\ c_Earithmic_E2B\ n))$

Definition 13 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_EEnum_EEnum.V0x$.

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealx_Ereal^{ty_EEnum_EEnum})^{ty_Erealx_Ereal}) \tag{11}$$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \tag{12}$$

Let $c_Erealx_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealx_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealx_Ereal}) \tag{13}$$

Definition 14 We define c_Emin_E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ \lambda x.x)$ of type $\iota \Rightarrow \iota$.

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Ereal_2Epow \\
& V0x) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Etransc_2Esqrt \\
& V0x))))))
\end{aligned} \tag{24}$$

Theorem 1

$$\begin{aligned}
& (\forall V0z \in (ty_2Epair_2Eprod ty_2Erealax_2Ereal ty_2Erealax_2Ereal). \\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap c_2Ecomplex_2Emodu V0z))))
\end{aligned}$$