

thm\_2Econtainer\_2EBAG\_\_TO\_\_LIST\_\_IND  
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October 26, 2020

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Ebag\_2EBAG\_CARD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebag\_2EBAG\_CARD\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{A\_27a})}) \tag{2}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_T$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ P))$

**Definition 4** We define  $c\_2Ebool\_2E\_F$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{5}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_2Ebag\_2EBAG\_INN$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_2Ebag\_2ESUB\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b1 \in$

**Definition 17** We define  $c\_2Ebag\_2EPSUB\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b1 \in$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (6)$$

Let  $c\_2Ebag\_2EBAG\_CHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Ebag\_2EBAG\_CHOICE\ A\_27a \in (A\_27a)^{(ty\_2Enum\_2Enum^{A\_27a})} \quad (7)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Elist\_2Elist\ A0) \quad (8)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (9)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 19** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 20** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a})$

**Definition 21** We define  $c\_Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a) (c\_Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a)) (c\_Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27a))$ .  
Let  $c\_Eenum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_2EZERO\_REP \in \omega \tag{11}$$

**Definition 22** We define  $c\_Eenum\_2E0$  to be  $(ap c\_Eenum\_2EABS\_num c\_Eenum\_2EZERO\_REP)$ .

**Definition 23** We define  $c\_Ebag\_2EEMPTY\_BAG$  to be  $\lambda A\_27a : \iota.(ap (c\_Ecombin\_2EK ty\_2Eenum\_2E0 A\_27a))$ .

**Definition 24** We define  $c\_Earithmic\_2EZERO$  to be  $c\_Eenum\_2E0$ .

Let  $c\_Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmic\_2E\_2B \in ((ty\_2Eenum\_2Eenum^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum}) \tag{12}$$

**Definition 25** We define  $c\_Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Eenum\_2Eenum.(ap (ap c\_Earithmic\_2E\_2B V0n))$ .

**Definition 26** We define  $c\_Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Eenum\_2Eenum.V0x$ .

**Definition 27** We define  $c\_Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Eenum\_2Eenum A\_27a)$ .

**Definition 28** We define  $c\_Ebag\_2EEL\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.(ap (ap (c\_Ebag\_2EBAG\_INSERT A\_27a) V0e))$ .

Let  $c\_Earithmic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmic\_2E\_2D \in ((ty\_2Eenum\_2Eenum^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum}) \tag{13}$$

**Definition 29** We define  $c\_Ebag\_2EBAG\_DIFF$  to be  $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Eenum\_2Eenum^{A\_27a}).\lambda V1b \in (ty\_2Eenum\_2Eenum^{A\_27a}).(ap (c\_Ebag\_2EBAG\_INSERT A\_27a) V0b1)$ .

**Definition 30** We define  $c\_Ebag\_2EBAG\_REST$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Eenum\_2Eenum^{A\_27a}).(ap (c\_Ebag\_2EBAG\_INSERT A\_27a) V0b)$ .

**Definition 31** We define  $c\_Ebag\_2EFINITE\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Eenum\_2Eenum^{A\_27a}).(ap (c\_Ebag\_2EBAG\_REST A\_27a) V0b)$ .

**Definition 32** We define  $c\_ERelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_Ebool\_2E21 A\_27a) V0R)$ .

**Definition 33** We define  $c\_ERelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1R \in (2^{A\_27a}).(ap (c\_ERelation\_2EWF A\_27a) V0f)$ .

**Definition 34** We define  $c\_ERelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b \in (A\_27b^{A\_27a}).(ap (c\_ERelation\_2ERESTRICT A\_27a) V0R)$ .

**Definition 35** We define  $c\_ERelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (2^{A\_27a}).(ap (c\_ERelation\_2ETC A\_27a) V0R)$ .

**Definition 36** We define  $c\_ERelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (2^{A\_27a}).(ap (c\_ERelation\_2Eapprox A\_27a) V0R)$ .

**Definition 37** We define  $c\_ERelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M \in (2^{A\_27a}).(ap (c\_ERelation\_2Ethe\_fun A\_27a) V0R)$ .

**Definition 38** We define  $c\_Econtainer\_2EBAG\_TO\_LIST$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_ERelation\_2EWFREC A\_27a) V0R))$ .

**Definition 39** We define  $c\_Erelation\_2Einv\_image$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27b})^{A\_27a}). \lambda V$

**Definition 40** We define  $c\_Eprim\_rec\_2Emeasure$  to be  $\lambda A\_27a : \iota. (ap (c\_Erelation\_2Einv\_image A\_27a t))$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & ((\neg(V0b = (c\_2Ebag\_2EEMPTY\_BAG A\_27a))) \Rightarrow (p (ap (ap (c\_2Ebag\_2EPSUB\_BAG \\ & A\_27a) (ap (c\_2Ebag\_2EBAG\_REST A\_27a) V0b)) V0b)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}). (((p (ap (c\_2Ebag\_2EFINITE\_BAG \\ & A\_27a) V1b2)) \wedge (p (ap (ap (c\_2Ebag\_2EPSUB\_BAG A\_27a) V0b1) V1b2))) \Rightarrow \\ & (p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap (c\_2Ebag\_2EBAG\_CARD A\_27a) \\ & V0b1)) (ap (c\_2Ebag\_2EBAG\_CARD A\_27a) V1b2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\forall V1x \in A\_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (29)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (30)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI\ A\_27a)\ V0x) = V0x)) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in (ty\_2Enum\_2Enum^{A\_27a}). (p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ (ap\ (c\_2Eprim\_rec\_2Emeasure\ A\_27a)\ V0m)))) \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ ((p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A\_27a}). \\ ((\forall V2x \in A\_27a. ((\forall V3y \in A\_27a. ((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow \\ (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A\_27a. (p\ (ap\ \\ V1P\ V4x)))))))))) \quad (34) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). (\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2y \in \\ A\_27a. (\forall V3z \in A\_27a. ((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap \\ (ap\ (c\_2Erelation\_2ERESTRICT\ A\_27a\ A\_27b)\ V0f)\ V1R)\ V3z)\ V2y) = \\ (ap\ V0f\ V2y)))))))))) \quad (35) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}). (\forall V1R \in ((2^{A\_27a})^{A\_27a}). \\ (\forall V2f \in (A\_27b^{A\_27a}). ((V2f = (ap\ (ap\ (c\_2Erelation\_2EWFREC \\ A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c\_2Erelation\_2EWF\ A\_27a)\ V1R)) \Rightarrow \\ (\forall V3x \in A\_27a. ((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c\_2Erelation\_2ERESTRICT \\ A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x)) V3x)))))))))) \quad (36) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (46)$$

### Theorem 1

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Enum\_2Enum^{A-27a})}). ((\forall V1bag \in (ty\_2Enum\_2Enum^{A-27a}).(((p (ap (c\_2Ebag\_2EFINITE\_BAG A.27a) V1bag)) \wedge (\neg(V1bag = (c\_2Ebag\_2EEMPTY\_BAG A.27a)))) \Rightarrow (p (ap V0P (ap (c\_2Ebag\_2EBAG\_REST A.27a) V1bag)))) \Rightarrow (p (ap V0P V1bag)))) \Rightarrow (\forall V2v \in (ty\_2Enum\_2Enum^{A-27a}).(p (ap V0P V2v))))$$