

# thm\_2Econtainer\_2EBAG\_\_TO\_\_LIST\_\_INV (TM- bRuCKHZENcgXxgy8WcAP6BEVSFxU214sa)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2ESUC\_REP m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_2Ebag\_2EBAG\_INN$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_2Ebag\_2ESUB\_BAG$  to be  $\lambda A\_27a : \iota. \lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). \lambda V1b2 \in$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 18** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 19** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B$

**Definition 20** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 21** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 22** We define  $c\_2Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1b \in (ty\_2Enum\_2Enum^{A\_27a})$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let  $c\_2Econtainer\_2ELIST\_TO\_BAG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Econtainer\_2ELIST\_TO\_BAG\ A\_27a \in ((ty\_2Enum\_2Enum^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{8}$$

Let  $c\_2Ebag\_2EBAG\_CHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebag\_2EBAG\_CHOICE\ A\_27a \in (A\_27a^{(ty\_2Enum\_2Enum^{A\_27a})}) \tag{9}$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (10)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (11)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (12)$$

**Definition 23** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 24** We define  $c\_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ecombin\_2EK\ ty\_2Enum\_2Enum\ A\_27a))$

**Definition 25** We define  $c\_2Ebag\_2EEL\_BAG$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. (ap\ (ap\ (c\_2Ebag\_2EBAG\_IN)))$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

**Definition 26** We define  $c\_2Ebag\_2EBAG\_DIFF$  to be  $\lambda A\_27a : \iota. \lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). \lambda V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$

**Definition 27** We define  $c\_2Ebag\_2EBAG\_REST$  to be  $\lambda A\_27a : \iota. \lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap\ (c\_2Ebag\_2EBAG\_IN))$

**Definition 28** We define  $c\_2Ebag\_2EFINITE\_BAG$  to be  $\lambda A\_27a : \iota. \lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap\ (c\_2Ebag\_2EBAG\_IN))$

**Definition 29** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 30** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A\_27a)))$

**Definition 31** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). (ap\ (c\_2Ebool\_2E\_21))$

**Definition 32** We define  $c\_2Erelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1g \in (A\_27b^{A\_27a}).$

**Definition 33** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda V1a \in A\_27a. \lambda V2b \in A\_27a.$

**Definition 34** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda V1M \in (A\_27b^{A\_27a}).$

**Definition 35** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda V1M \in (A\_27b^{A\_27a}).$

**Definition 36** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). \lambda V1M \in (A\_27b^{A\_27a}).$

**Definition 37** We define  $c\_2Econtainer\_2EBAG\_TO\_LIST$  to be  $\lambda A\_27a : \iota. (ap\ (ap\ (c\_2Erelation\_2EWFREC)))$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A_{.27a}}). \\ & ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V0b1)) \Rightarrow (\forall V1b2 \in ( \\ & ty\_2Enum\_2Enum^{A_{.27a}}). ((p\ (ap\ (ap\ (c\_2Ebag\_2ESUB\_BAG\ A_{.27a}) \\ & V1b2)\ V0b1)) \Rightarrow (p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V1b2)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A_{.27a}}). \\ & ((\neg(V0b = (c\_2Ebag\_2EEMPTY\_BAG\ A_{.27a}))) \Rightarrow (V0b = (ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT \\ & A_{.27a})\ (ap\ (c\_2Ebag\_2EBAG\_CHOICE\ A_{.27a})\ V0b))\ (ap\ (c\_2Ebag\_2EBAG\_REST \\ & A_{.27a})\ V0b)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A_{.27a}}). \\ & (p\ (ap\ (ap\ (c\_2Ebag\_2ESUB\_BAG\ A_{.27a})\ (ap\ (c\_2Ebag\_2EBAG\_REST \\ & A_{.27a})\ V0b))\ V0b))) \end{aligned} \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\
& A\_27a.(((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \\
& V0t1) \ V1t2) = V1t2))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in ( \\
& 2^{A\_27a}).((\forall V2x \in A\_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \vee (\forall V3x \in A\_27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\
& (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C))))))
\end{aligned} \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & (((ap (c.2Econtainer\_2ELIST\_TO\_BAG \\ A.27a) (c.2Elist\_2ENIL \ A.27a)) = (c.2Ebag\_2EEMPTY\_BAG \ A.27a)) \wedge \\ & (\forall V0h \in A.27a.(\forall V1t \in (ty.2Elist.2Elist \ A.27a). \\ & (ap (c.2Econtainer\_2ELIST\_TO\_BAG \ A.27a) (ap (ap (c.2Elist.2ECONS \\ A.27a) \ V0h) \ V1t)) = (ap (ap (c.2Ebag\_2EBAG\_INSERT \ A.27a) \ V0h) ( \\ & ap (c.2Econtainer\_2ELIST\_TO\_BAG \ A.27a) \ V1t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & (\forall V0bag \in (ty.2Enum.2Enum^{A.27a}). \\ ((p (ap (c.2Ebag\_2EFINITE\_BAG \ A.27a) \ V0bag)) \Rightarrow & ((ap (c.2Econtainer\_2EBAG\_TO\_LIST \\ A.27a) \ V0bag) = (ap (ap (ap (c.2Ebool\_2ECOND (ty.2Elist.2Elist \\ A.27a)) (ap (ap (c.2Emin.2E\_3D (ty.2Enum.2Enum^{A.27a}) \ V0bag) \\ (c.2Ebag\_2EEMPTY\_BAG \ A.27a))) (c.2Elist.2ENIL \ A.27a)) (ap ( \\ ap (c.2Elist.2ECONS \ A.27a) (ap (c.2Ebag\_2EBAG\_CHOICE \ A.27a) \\ V0bag)) (ap (c.2Econtainer\_2EBAG\_TO\_LIST \ A.27a) (ap (c.2Ebag\_2EBAG\_REST \\ A.27a) \ V0bag)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow & (\forall V0P \in (2^{(ty.2Enum.2Enum^{A.27a})}). \\ ((\forall V1bag \in (ty.2Enum.2Enum^{A.27a}).(((p (ap (c.2Ebag\_2EFINITE\_BAG \\ A.27a) \ V1bag)) \wedge \neg(V1bag = (c.2Ebag\_2EEMPTY\_BAG \ A.27a)))) \Rightarrow & ( \\ p (ap \ V0P (ap (c.2Ebag\_2EBAG\_REST \ A.27a) \ V1bag))) \Rightarrow & (p (ap \ V0P \ V1bag))) \Rightarrow \\ & (\forall V2v \in (ty.2Enum.2Enum^{A.27a}).(p (ap \ V0P \ V2v)))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (47)$$

### Theorem 1

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}). ((p (ap (c\_2Ebag\_2EFINITE\_BAG A\_27a) V0b)) \Rightarrow ((ap (c\_2Econtainer\_2ELIST\_TO\_BAG A\_27a) (ap (c\_2Econtainer\_2EBAG\_TO\_LIST A\_27a) V0b)) = V0b)))$$