

thm_2Econtainer_2EBAG_TO_LIST_THM (TMQumxhhb5Gjiwd34ZmReDjhJbNrZM3G19i)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Ebag_2EBAG_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ebag_2EBAG_CARD\ A_27a \in (ty_2Enum_2Enum^{(ty_2Enum_2Enum^{A_27a})}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (V0m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota)$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ (V0P)))\ (c_2Emin_2E_40\ (V0P)))$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Emin_2E_40\ (V0m))$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Emin_2E_40\ (V0m))$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (V2t \in 2))))$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Emin_2E_40\ (V0m))$

Definition 15 We define $c_2Ebag_2EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum.(c_2Emin_2E_40\ (V0e))$

Definition 16 We define $c_2Ebag_2ESUB_BAG$ to be $\lambda A_27a : \iota.\lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}).\lambda V1b1 \in (ty_2Enum_2Enum^{A_27a}).(c_2Emin_2E_40\ (V0b1))$

Definition 17 We define $c_2Ebag_2EPSUB_BAG$ to be $\lambda A_27a : \iota.\lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}).\lambda V1b1 \in (ty_2Enum_2Enum^{A_27a}).(c_2Emin_2E_40\ (V0b1))$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (6)$$

Let $c_2Ebag_2EBAG_CHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Ebag_2EBAG_CHOICE\ A_27a \in (A_27a^{(ty_2Enum_2Enum^{A_27a})}) \quad (7)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (9)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (10)$$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.((V0t = t1) \wedge (V1t1 = t2)))))$

Definition 19 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.(V0x = y)))$

Definition 20 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 21 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a) (A_27a^{A-27a})) A_27a)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Definition 22 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 23 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota. (ap (c_2Ecombin_2EK ty_2Enum_2Enum) A_27a)$

Definition 24 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 25 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B) V0n)$

Definition 26 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 27 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2Enum) A_27a$

Definition 28 We define $c_2Ebag_2EEL_BAG$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. (ap (ap (c_2Ebag_2EBAG_IN)) V0e) A_27a)$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 29 We define $c_2Ebag_2EBAG_DIFF$ to be $\lambda A_27a : \iota. \lambda V0b1 \in (ty_2Enum_2Enum^{A-27a}). \lambda V1b2 \in (ty_2Enum_2Enum^{A-27a}) A_27a$

Definition 30 We define $c_2Ebag_2EBAG_REST$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A-27a}). (ap (c_2Ebag_2EBAG_DIFF) V0b) A_27a$

Definition 31 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A-27a}). (ap (c_2Ebag_2EBAG_REST) V0b) A_27a$

Definition 32 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). (ap (c_2Ebool_2E_21) V0R) A_27a$

Definition 33 We define $c_2Erelation_2ERESTRIC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A-27a}). \lambda V1M \in (A_27b^{A-27a}) A_27a$

Definition 34 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V1a \in A_27a. \lambda V2b \in (A_27b^{A-27a}) A_27a$

Definition 35 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V1M \in (A_27b^{A-27a}) A_27a$

Definition 36 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V1M \in (A_27b^{A-27a}) A_27a$

Definition 37 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A-27a})^{A-27a}). \lambda V1M \in (A_27b^{A-27a}) A_27a$

Definition 38 We define $c_2Econtainer_2EBAG_TO_LIST$ to be $\lambda A_27a : \iota. (ap (ap (c_2Erelation_2EWFREC) V0R) A_27a)$

Definition 39 We define $c_2Erelation_2Einv_image$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27b}). \lambda V$

Definition 40 We define $c_2Eprim_rec_2Emeasure$ to be $\lambda A_27a : \iota. (ap (c_2Erelation_2Einv_image A_27a t_0) t_1)$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & ((\neg(V0b = (c_2Ebag_2EEMPTY_BAG A_27a))) \Rightarrow (p (ap (ap (c_2Ebag_2EPSUB_BAG \\ & A_27a) (ap (c_2Ebag_2EBAG_REST A_27a) V0b)) V0b)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b1 \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1b2 \in (ty_2Enum_2Enum^{A_27a}). (((p (ap (c_2Ebag_2EFINITE_BAG \\ & A_27a) V1b2)) \wedge (p (ap (ap (c_2Ebag_2EPSUB_BAG A_27a) V0b1) V1b2))) \Rightarrow \\ & (p (ap (ap (c_2Eprim_rec_2E_3C (ap (c_2Ebag_2EBAG_CARD A_27a) \\ & V0b1)) (ap (c_2Ebag_2EBAG_CARD A_27a) V1b2))))))) \end{aligned} \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. (\forall V1y \in A_{\text{27a}}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t))) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0t1 \in A_{\text{27a}}. (\forall V1t2 \in A_{\text{27a}}. ((ap (ap (ap (c_{\text{2Ebool_2ECOND}} A_{\text{27a}}) c_{\text{2Ebool_2ET}}) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_{\text{2Ebool_2ECOND}} A_{\text{27a}}) c_{\text{2Ebool_2EF}}) V0t1) V1t2) = V1t2)))))) \\ & \end{aligned} \quad (25)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}}). ((\neg(\forall V1x \in A_{\text{27a}}. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_{\text{27a}}. (\neg(p (ap V0P V2x))))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_{\text{27}} \in 2. (\forall V2y \in 2. (\forall V3y_{\text{27}} \in 2. (((p V0x) \Leftrightarrow (p V1x_{\text{27}})) \wedge ((p V1x_{\text{27}}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{\text{27}})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{\text{27}}) \Rightarrow (p V3y_{\text{27}}))))))) \\ & \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27a}) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_{.2Ebool_2ECOND}\ A_{.27a})\ V1Q)\ V3x_{.27}) \\ & V5y_{.27}))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0x \in A_{.27a}.((ap\ (c_{.2Ecombin_2EI}\ \\ & A_{.27a})\ V0x) = V0x)) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0m \in (ty_{.2Enum_2Enum}^{A_{.27a}}). \\ & (p\ (ap\ (c_{.2Erelation_2EWF}\ A_{.27a})\ (ap\ (c_{.2Eprim_rec_2Emeasure}\ \\ & A_{.27a})\ V0m)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0R \in ((2^{A_{.27a}})^{A_{.27a}}). \\ & ((p\ (ap\ (c_{.2Erelation_2EWF}\ A_{.27a})\ V0R)) \Rightarrow (\forall V1P \in (2^{A_{.27a}}). \\ & ((\forall V2x \in A_{.27a}.((\forall V3y \in A_{.27a}.((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow \\ & (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A_{.27a}.(p\ (ap\ \\ & V1P\ V4x))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \\ & (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1R \in ((2^{A_{.27a}})^{A_{.27a}}).(\forall V2y \in \\ & A_{.27a}.(\forall V3z \in A_{.27a}.((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap\ \\ & (ap\ (c_{.2Erelation_2ERESTRICT}\ A_{.27a}\ A_{.27b})\ V0f)\ V1R)\ V3z)\ V2y) = \\ & (ap\ V0f\ V2y))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \\ & (\forall V0M \in ((A_{.27b}^{A_{.27a}})^{(A_{.27b}^{A_{.27a}})}).(\forall V1R \in ((2^{A_{.27a}})^{A_{.27a}}). \\ & (\forall V2f \in (A_{.27b}^{A_{.27a}}).((V2f = (ap\ (ap\ (c_{.2Erelation_2EWFREC}\ \\ & A_{.27a}\ A_{.27b})\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c_{.2Erelation_2EWF}\ A_{.27a})\ V1R)) \Rightarrow \\ & (\forall V3x \in A_{.27a}.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (c_{.2Erelation_2ERESTRICT}\ \\ & A_{.27a}\ A_{.27b})\ V2f)\ V1R)\ V3x))))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow ((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge ((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee ((\neg(p V1q) \vee (p V2r)) \vee ((\neg(p V1q) \vee (p V0p)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow ((p V0p) \vee ((\neg(p V1q) \wedge (p V2r))) \wedge ((p V1q) \vee ((\neg(p V0p) \wedge (p V2r)) \vee ((\neg(p V0p) \wedge (p V2r)) \vee ((\neg(p V0p) \wedge (p V1q)))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow ((p V0p) \vee ((\neg(p V1q) \vee (p V2r))) \wedge ((p V0p) \vee ((\neg(p V2r) \vee (p V1q)) \vee ((\neg(p V2r) \vee (p V1q)) \vee ((\neg(p V0p) \vee (p V2r)))))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow ((p V0p) \vee ((\neg(p V1q) \Rightarrow (p V2r))) \wedge ((p V0p) \vee ((\neg(p V2r) \Rightarrow (p V1q)) \vee ((\neg(p V2r) \Rightarrow (p V1q)) \vee ((\neg(p V0p) \Rightarrow (p V2r)))))))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V0p) \vee (\neg(p V1q))))))) \quad (47)$$

Theorem 1
$$\begin{aligned} & \forall A_{_27a}. \text{nonempty } A_{_27a} \Rightarrow (\forall V0bag \in (ty_2Enum_2Enum^{A_27a}). \\ & ((p (ap (c_2Ebag_2EFINITE_BAG A_{_27a}) V0bag)) \Rightarrow ((ap (c_2Econtainer_2EBAG_TO_LIST \\ & \quad A_{_27a}) V0bag) = (ap (ap (ap (c_2Ebool_2ECOND (ty_2Elist_2Elist \\ & \quad A_{_27a})) (ap (ap (c_2Emin_2E_3D (ty_2Enum_2Enum^{A_27a})) V0bag) \\ & \quad (c_2Ebag_2EEMPTY_BAG A_{_27a}))) (c_2Elist_2ENIL A_{_27a})) (ap (\\ & \quad ap (c_2Elist_2ECONS A_{_27a}) (ap (c_2Ebag_2EBAG_CHOICE A_{_27a}) \\ & \quad V0bag)) (ap (c_2Econtainer_2EBAG_TO_LIST A_{_27a}) (ap (c_2Ebag_2EBAG_REST \\ & \quad A_{_27a}) V0bag))))))) \end{aligned}$$