

thm_2Econtainer_2EEVERY__LIST__TO__BAG (TMKRemhQsy1LvkJaai1Gi7PRqaMudpN2JiP)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define `c_2Earithmic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2EBIT1 V0n) V0n)$.

Definition 8 We define `c_2Earithmic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E21) V0t)$.

Definition 12 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 13 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 V0P) V0P)))$.

Definition 15 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Definition 16 We define `c_2Earithmic_2E_3E` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Definition 17 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 18 We define `c_2Earithmic_2E_3E_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$.

Definition 19 We define `c_2Ebag_2EBAG_INN` to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum.V0e$.

Definition 20 We define `c_2Ebag_2EBAG_IN` to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V0e$.

Definition 21 We define `c_2Ebag_2EBAG_EVERY` to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V0P$.

Definition 22 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2) V1t1) V0t)$.

Definition 23 We define `c_2Ebag_2EBAG_INSERT` to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V0e$.

Definition 24 We define `c_2Ecombin_2EK` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$.

Definition 25 We define `c_2Ebag_2EEMPTY_BAG` to be $\lambda A_27a : \iota.(ap (c_2Ecombin_2EK ty_2Enum_2Enum^{A_27a}) V0e)$.

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (7)$$

Let `c_2Econtainer_2ELIST_TO_BAG` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Econtainer_2ELIST_TO_BAG A_27a \in ((ty_2Enum_2Enum^{A_27a})(ty_2Elist_2Elist A_27a)) \quad (8)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (9)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (10)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0P \in (2^{A_27a}).(p\ (ap\ (ap\ (c_2Ebag_2EBAG_EVERY\ A_27a) \\ & V0P)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)))) \wedge (\forall V1P \in (2^{A_27b}). \\ & (\forall V2e \in A_27b.(\forall V3b \in (ty_2Enum_2Enum^{A_27b}).((p \\ & (ap\ (ap\ (c_2Ebag_2EBAG_EVERY\ A_27b)\ V1P)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\ & A_27b)\ V2e)\ V3b)))) \Leftrightarrow ((p\ (ap\ V1P\ V2e)) \wedge (p\ (ap\ (ap\ (c_2Ebag_2EBAG_EVERY \\ & A_27b)\ V1P)\ V3b)))))) \end{aligned} \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Econtainer_2ELIST_TO_BAG \\ & A_27a)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Ebag_2EEMPTY_BAG\ A_27a)) \wedge \\ & (\forall V0h \in A_27a.(\forall V1t \in (ty_2Elist_2Elist\ A_27a).(\\ & (ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27a)\ V0h)\ (\\ & ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ V1t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((p\ (ap \\
& (ap\ (c_2Elist_2EEVERY\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ (ap\ (c_2Elist_2EEVERY\ A_27a)\ V1P)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c_2Elist_2EEVERY \\
& A_27a)\ V1P)\ V3t))))))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\
& (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\
& A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\
& c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\
& A_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1ls \in \\
& (ty_2Elist_2Elist\ A_27a).((p\ (ap\ (ap\ (c_2Ebag_2EBAG_EVERY\ A_27a) \\
& V0P)\ (ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ V1ls))) \Leftrightarrow (p\ (ap\ (\\
& (ap\ (c_2Elist_2EEVERY\ A_27a)\ V0P)\ V1ls))))))
\end{aligned}$$