

# thm\_2Econtainer\_2ELIST\_\_ELEM\_\_COUNT\_\_LIST\_\_TO\_\_BAG (TMKKTogsQBU5s7z9xohGzNUaYHinksThS9YL)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t))))$

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t))))$

**Definition 13** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge P x)$ ) of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t)))$

**Definition 15** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 16** We define  $c\_2Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum^A\_27a)$

**Definition 17** We define  $c\_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ecombin\_2EK ty\_2Enum\_2Enum^A\_27a))$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (7)$$

Let  $c\_2Econtainer\_2ELIST\_TO\_BAG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Econtainer\_2ELIST\_TO\_BAG A\_27a \in ((ty\_2Enum\_2Enum^{A\_27a}(ty\_2Elist\_2Elist A\_27a))A\_27a) \quad (8)$$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_5C\_2F))$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))A\_27a) \quad (9)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)(ty\_2Elist\_2Elist A\_27a))(ty\_2Elist\_2Elist A\_27a)) \quad (10)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (11)$$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (12)$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (13)$$

**Definition 19** We define  $c\_2Erich\_list\_2ELIST\_ELEM\_COUNT$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1l \in (ty$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee \neg(p\ V0t))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27)))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (ap\ (c\_2Ecombin\_2EK\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (((ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG \\ & A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27a)) \wedge \\ & (\forall V0h \in A\_27a. (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a). ( \\ & (ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A\_27a)\ V0h)\ ( \\ & ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG\ A\_27a)\ V1t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A.27a)}), \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V1t))) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& \quad c\_2Elist\_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& \quad A.27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow ((\forall V0e \in A.27a. \\
& \quad ((ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27a)\ V0e)\ (c\_2Elist\_2ENIL \\
& \quad \quad A.27a)) = c\_2Enum\_2E0)) \wedge ((\forall V1e \in A.27b. (\forall V2l1 \in ( \\
& \quad \quad ty\_2Elist\_2Elist\ A.27b). (\forall V3l2 \in (ty\_2Elist\_2Elist\ A.27b). \\
& \quad \quad ((ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27b)\ V1e)\ (ap \\
& \quad \quad (ap\ (c\_2Elist\_2EAPPEND\ A.27b)\ V2l1)\ V3l2)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& \quad \quad (ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27b)\ V1e)\ V2l1)) \\
& \quad \quad (ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27b)\ V1e)\ V3l2)))))) \wedge \\
& \quad ((\forall V4e \in A.27c. (\forall V5h \in A.27c. (\forall V6l \in (ty\_2Elist\_2Elist \\
& \quad \quad A.27c). ((V5h = V4e) \Rightarrow ((ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT \\
& \quad \quad A.27c)\ V4e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27c)\ V5h)\ V6l)) = (ap\ c\_2Enum\_2ESUC \\
& \quad \quad (ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27c)\ V4e)\ V6l)))))) \wedge \\
& \quad (\forall V7e \in A.27d. (\forall V8h \in A.27d. (\forall V9l \in (ty\_2Elist\_2Elist \\
& \quad \quad A.27d). ((\neg(V8h = V7e)) \Rightarrow ((ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT \\
& \quad \quad A.27d)\ V7e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A.27d)\ V8h)\ V9l)) = (ap\ (ap\ ( \\
& \quad \quad c\_2Erich\_list\_2ELIST\_ELEM\_COUNT\ A.27d)\ V7e)\ V9l)))))))))
\end{aligned} \tag{29}$$

**Theorem 1**

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0e \in A.27a. (\forall V1ls \in \\
& \quad (ty\_2Elist\_2Elist\ A.27a). ((ap\ (ap\ (c\_2Erich\_list\_2ELIST\_ELEM\_COUNT \\
& \quad \quad A.27a)\ V0e)\ V1ls) = (ap\ (ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG\ A.27a) \\
& \quad \quad \quad V1ls)\ V0e))))))
\end{aligned}$$