

thm_2Econtainer_2ELIST_TO_BAG_EQ_EMPTY
 (TMYESTLgWUo-
 faXf6mT7rRSRnLkxRhCfW7bG)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 6 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in$

Definition 14 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2E$

Definition 15 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 16 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap\ (c_2Ecombin_2EK\ ty_2Enum_2En$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty_2Elist_2Elist\ A0) \quad (7)$$

Let $c_2Econtainer_2ELIST_TO_BAG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Econtainer_2ELIST_TO_BAG \quad (8)$$

$$A_27a \in ((ty_2Enum_2Enum^{A_27a})^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}$$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P))$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ECONS \quad (9)$$

$$A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})^{A_27a}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Elist_2ENIL \quad (10)$$

$$A_27a \in (ty_2Elist_2Elist\ A_27a)$$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1b \in \\ (ty_2Enum_2Enum^{A_{27a}}).(\neg((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a}) \\ V0x)\ V1b) = (c_2Ebag_2EEMPTY_BAG\ A_{27a})))))) \end{aligned} \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((V0x = V0x) \Leftrightarrow \\ True)) \quad (16)$$

Assume the following.

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow (\forall V0x \in A_{27a}.(\forall V1y \in \\ A_{27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (18)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (((ap\ (c_2Econtainer_2ELIST_TO_BAG\ \\ A_{27a})\ (c_2Elist_2ENIL\ A_{27a})) = (c_2Ebag_2EEMPTY_BAG\ A_{27a})) \wedge \\ & ((\forall V0h \in A_{27a}.(\forall V1t \in (ty_2Elist_2Elist\ A_{27a}).(\\ (ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_{27a})\ (ap\ (ap\ (c_2Elist_2ECONS\ \\ A_{27a})\ V0h)\ V1t)) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V0h)\ (\\ ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_{27a})\ V1t))))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0l \in (\text{ty_2Elist_2Elist } \\ A_27a).((V0l = (c_2Elist_2ENIL A_27a)) \vee (\exists V1h \in A_27a. \\ \exists V2t \in (\text{ty_2Elist_2Elist } A_27a). (V0l = (ap (ap (c_2Elist_2ECONS \\ A_27a) V1h) V2t))))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0a1 \in (\text{ty_2Elist_2Elist } \\ A_27a).(\forall V1a0 \in A_27a.(\neg((c_2Elist_2ENIL A_27a) = (ap (\\ ap (c_2Elist_2ECONS A_27a) V1a0) V0a1)))))) \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (\forall V0l \in (\text{ty_2Elist_2Elist } \\ A_27a).(((ap (c_2Econtainer_2ELIST_TO_BAG A_27a) V0l) = (c_2Ebag_2EMPTY_BAG \\ A_27a)) \Leftrightarrow (V0l = (c_2Elist_2ENIL A_27a)))) \end{aligned}$$