

thm\_2Econtainer\_2ELIST\_TO\_BAG\_\_alt  
 (TMN2CD6gRi47LwF42P4N7StFZsqquBhQrhU)

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Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2ABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2ABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2ABS\_num\ c\_2Enum\_2ZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2EBIT1\ n)\ V)$

**Definition 8** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$ .

**Definition 10** We define  $c_{\text{2Emin\_3D\_3D\_3E}}$  to be  $\lambda P \in 2.\lambda Q \in 2.\text{inj\_o} (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 12** We define  $c_{\text{2Emin\_2E\_40}}$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then} \ (\lambda x. x \in A \wedge \text{of type } \iota \Rightarrow \iota)$ .

**Definition 13** We define  $c_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.(\dots)))$

**Definition 14** We define  $c\_EBAG\_EBAG\_INSERT$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1b : (ty\_2Enum\_2E$

**Definition 15** We define  $c_2$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

**Definition 16** We define  $c_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A.\lambda 27a:\iota.(ap(c_2Ecombin_2EK\ ty\_2Enum\_2En$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow \text{nonempty} (\text{ty\_2Elist\_2Elist } A) \quad (7)$$

Let  $c\_2Econtainer\_2ELIST\_TO\_BAG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2E\text{container}\_2ELIST\_TO\_BAG \\ A\_27a \in ((ty\_2Enum\_2Enum^A\_{27a})(ty\_2Elist\_2Elist\_{A\_27a})) \quad (8)$$

Let  $c\_Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (\text{ty\_2Enum\_2Enum}^{(ty\_2Elist\_2Elist\ A\_27a)})$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A \_27a. nonempty\ A \_27a \Rightarrow c\_2Elist\_2EFILTER\ A \_27a \in (((ty\_2Elist\_2Elist\ A \_27a)^{(ty\_2Elist\_2Elist\ A \_27a)})^{(2^{A \_27a})}) \quad (10)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2\text{Elist\_2ECONS } A_27a \in (((ty\_2\text{Elist\_2Elist } A_27a)^{(ty\_2\text{Elist\_2Elist } A_27a)})^{A_27a}) \quad (11)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. \text{nonempty } A \Rightarrow c_2Elist_2ENIL \quad A \in (ty_2Elist_2Elist \\ A) \quad (12)$$

Let  $c_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (13)$$

Let  $c_2$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 17** We define  $c_2Eb0l_2E_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Eb0l_2E_7E))$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A.\_27a : \iota.(\lambda V0P \in (2^A\_{-}27a)).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 19** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 20** We define  $c_2Earthmetic_2E_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 21** We define  $c_{\text{CBool}} \_ 2E \_ 5C \_ 2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_{\text{CBool}} \_ 2E \_ 21 \_ 2) (\lambda V2t \in$

**Definition 22** We define  $c\_2Earthmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 23** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (ap\ (c\_2Ebool\_2B$

Let  $c_2$  be given. Assume the following.

*c\_2Earithmetic\_2EEXP*  $\in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^*$

Let  $c_2$  be given. Assume the following. (13)

Let  $\mathbf{S} \in \mathbb{R}^{n \times m}$  be given. Assume the following.

Let  $c_2$  be arithmetic. Then  $c_2$  is a linear function of  $\lambda$ , as follows:

Let  $c_2$  be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

**Definition 24** We define  $c\_2Enumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2ESUC\ (ap$

**Definition 25** We define  $c_2E\text{Enum}_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 26** We define  $c\_2Earthmetic\_2EBIT2$  to be  $\lambda Vn \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earthmetic\ n\ V)\ n)$

**Definition 27** We define  $c_2$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum.$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (18)$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) V2p)))))) \\
 & \tag{19}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
 & \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
 & \quad (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
 & \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge \\
 & \quad ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m) V1n) = (ap \\
 & \quad (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
 & \quad V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
 & \quad V0m) V1n)))))))))) \\
 & \tag{20}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \quad (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \\
 & \tag{21}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \quad \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
 & \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
 & \quad V0m) V2p)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \\
 & \tag{22}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
 & \quad (\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
 & \quad V1n)) V0m)))))) \\
 & \tag{23}
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap \\
 & \quad c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
 & \quad c\_2Earithmetic\_2EZERO))) V0n))) \\
 & \tag{24}
 \end{aligned}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow ((\text{c\_2Eb} \_2EEMPTY\_BAG A\_27a) = (\lambda V0x \in A\_27a.\text{c\_2Enum\_2E0})) \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (29)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge (p V1t2) \wedge (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (32)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (33)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ & A\_27a.((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ & V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (((ap (c\_2Econtainer\_2ELIST\_TO\_BAG \\ & A\_27a) (c\_2Elist\_2ENIL A\_27a)) = (c\_2Ebag\_2EEMPTY\_BAG A\_27a)) \wedge \\ & (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist A\_27a). \\ & (ap (c\_2Econtainer\_2ELIST\_TO\_BAG A\_27a) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V0h) V1t)) = (ap (ap (c\_2Ebag\_2EBAG\_INSERT A\_27a) V0h) \\ & (ap (c\_2Econtainer\_2ELIST\_TO\_BAG A\_27a) V1t)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
 \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (((ap\ (c\_2Elist\_2ELENGTH\ A_{27a}) \\
 & (c\_2Elist\_2ENIL\ A_{27a})) = c\_2Enum\_2E0) \wedge (\forall V0h \in A_{27a}.( \\
 & \forall V1t \in (ty\_2Elist\_2Elist\ A_{27a}).((ap\ (c\_2Elist\_2ELENGTH \\
 & A_{27a})\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V0h)\ V1t)) = (ap\ c\_2Enum\_2ESUC \\
 & (ap\ (c\_2Elist\_2ELENGTH\ A_{27a})\ V1t)))))))
 \end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
 \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((\forall V0P \in (2^{A_{27a}}).((ap\ ( \\
 & ap\ (c\_2Elist\_2EFILTER\ A_{27a})\ V0P)\ (c\_2Elist\_2ENIL\ A_{27a})) = (c\_2Elist\_2ENIL \\
 & A_{27a}))) \wedge (\forall V1P \in (2^{A_{27a}}).(\forall V2h \in A_{27a}.(\forall V3t \in \\
 & (ty\_2Elist\_2Elist\ A_{27a}).((ap\ (ap\ (c\_2Elist\_2EFILTER\ A_{27a}) \\
 & V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A_{27a})\ V2h)\ V3t)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
 & (ty\_2Elist\_2Elist\ A_{27a}))\ (ap\ V1P\ V2h))\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
 & A_{27a})\ V2h)\ (ap\ (ap\ (c\_2Elist\_2EFILTER\ A_{27a})\ V1P)\ V3t)))\ (ap\ (ap\ \\
 & (c\_2Elist\_2EFILTER\ A_{27a})\ V1P)\ V3t)))))))
 \end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
 \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A_{27a})}). \\
 & (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A_{27a}))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
 & A_{27a}).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_{27a}.(p\ (ap\ V0P\ (ap\ (ap \\
 & c\_2Elist\_2ECONS\ A_{27a})\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
 & A_{27a}).(p\ (ap\ V0P\ V3l))))))
 \end{aligned} \tag{46}$$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B c_2Enum\_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V1n) c_2Enum\_2E0) = V1n)) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2B V3m) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2EiZ (ap (ap c_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge (\forall V4n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A c_2Enum\_2E0) V4n) = c_2Enum\_2E0)) \wedge (\forall V5n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A V5n) c_2Enum\_2E0) = c_2Enum\_2E0)) \wedge (\forall V6n \in ty\_2Enum\_2Enum. (\forall V7m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2A (ap c_2Earithmetic\_2ENUMERAL V6n)) (ap c_2Earithmetic\_2ENUMERAL V7m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2A V6n) V7m)))))) \wedge (\forall V8n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D c_2Enum\_2E0) V8n) = c_2Enum\_2E0)) \wedge (\forall V9n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D V9n) c_2Enum\_2E0) = V9n)) \wedge (\forall V10n \in ty\_2Enum\_2Enum. (\forall V11m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2E\_2D (ap c_2Earithmetic\_2ENUMERAL V10n)) (ap c_2Earithmetic\_2ENUMERAL V11m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2E\_2D V10n) V11m)))))) \wedge (\forall V12n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 V12n)))) = c_2Enum\_2E0)) \wedge (\forall V13n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT2 V13n)))) = c_2Enum\_2E0)) \wedge (\forall V14n \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP V14n) c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))))) \wedge (\forall V15n \in ty\_2Enum\_2Enum. (\forall V16m \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic\_2EEEXP (ap c_2Earithmetic\_2ENUMERAL V15n)) (ap c_2Earithmetic\_2ENUMERAL V16m)) = (ap c_2Earithmetic\_2ENUMERAL (ap (ap c_2Earithmetic\_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum\_2ESUC c_2Enum\_2E0) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Earithmetic\_2EBIT1 c_2Earithmetic\_2EZERO)))) \wedge (\forall V17n \in ty\_2Enum\_2Enum. ((ap c_2Enum\_2ESUC (ap c_2Earithmetic\_2ENUMERAL V17n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Enum\_2ESUC V17n)))))) \wedge (((ap c_2Eprim\_rec\_2EPRE c_2Enum\_2E0) = c_2Enum\_2E0) \wedge (\forall V18n \in ty\_2Enum\_2Enum. ((ap c_2Eprim\_rec\_2EPRE (ap c_2Earithmetic\_2ENUMERAL V18n)) = (ap c_2Earithmetic\_2ENUMERAL (ap c_2Eprim\_rec\_2EPRE V18n)))))) \wedge (\forall V19n \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V19n) = c_2Enum\_2E0) \Leftrightarrow (V19n = c_2Earithmetic\_2EZERO))) \wedge (\forall V20n \in ty\_2Enum\_2Enum. ((c_2Enum\_2E0 = (ap c_2Earithmetic\_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic\_2EZERO))) \wedge (\forall V21n \in ty\_2Enum\_2Enum. ((\forall V22m \in ty\_2Enum\_2Enum. (((ap c_2Earithmetic\_2ENUMERAL V21n) = (ap c_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C V23n) c_2Enum\_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Enum\_2E0) (ap c_2Earithmetic\_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2EZERO) V24n)))) \wedge (\forall V25n \in ty\_2Enum\_2Enum. (\forall V26m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Eprim\_rec\_2E\_3C c_2Earithmetic\_2ENUMERAL V25n) (ap c_2Earithmetic\_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim\_rec\_2E\_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3E c_2Earithmetic\_2ENUMERAL V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V29n)) \Leftrightarrow True))) \wedge (\forall V30m \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3E c_2Enum\_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic\_2E\_3E c_2Earithmetic\_2ENUMERAL V30m)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty\_2Enum\_2Enum. ((p (ap (ap c_2Earithmetic\_2E\_3D c_2Enum\_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))))
 \end{aligned} \tag{58}$$

### Theorem 1

$$\begin{aligned}
 & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0l \in (\text{ty\_2Elist\_2Elist} \\
 & A\_27a).(\forall V1x \in A\_27a.((\text{ap } (\text{ap } (\text{c\_2Econtainer\_2ELIST\_TO\_BAG} \\
 & A\_27a) V0l) V1x) = (\text{ap } (\text{c\_2Elist\_2ELENGTH } A\_27a) (\text{ap } (\text{ap } (\text{c\_2Elist\_2EFILTER} \\
 & A\_27a) (\text{ap } (\text{c\_2Emin\_2E\_3D } A\_27a) V1x)) V0l))))))
 \end{aligned}$$