

thm_2Econtainer_2EPERM_LIST_TO_BAG
 (TMaNQxehN-
 vyk53q6nT2gwbaW7RakMAp9Mfx)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Econtainer_2ELIST_TO_BAG : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Econtainer_2ELIST_TO_BAG\ A_27a \in ((ty_2Enum_2Enum^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \tag{4}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Esorting_2EPERM$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist\ A_27a).\lambda V1L2 \in (ty_2Elist_2Elist\ A_27a).$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{5}$$

Let $c_2Erich_list_2EREPLICATE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Erich_list_2EREPLICATE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{A_27a})^{ty_2Enum_2Enum} \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((\forall V2x \in A_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x))) \Rightarrow (V0f = V1g)))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ V0l)\ V1x) = (ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V1x))\ V0l)))) \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x))\ V1l) = (ap\ (ap\ (c_2Erich_list_2EREPLICATE\ A_27a)\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a)\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V0x))\ V1l)))\ V0x)))) \quad (13)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (((ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ V0l1) = (ap\ (c_2Econtainer_2ELIST_TO_BAG\ A_27a)\ V1l2)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Esorting_2EPERM\ A_27a)\ V0l1)\ V1l2))))))$$