

thm_2Econtainer_2ESET__TO__LIST__IND (TMJ9BHpCJ227wRqdiJFh9rUYRNJeJBfQp)

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Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in (A_27a)^{(2^{A_27a})} \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ (\lambda V1f \in 2.V1f)\ V0x)))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Epred_set_2EEEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E_2F)$.

Definition 6 We define $c_2Ebool_2E_2IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)\ V1t2)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t)\ V1t2)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (3)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (4)$$

Definition 11 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2).(ap (ap c_2Emin_2E_3D_3D_3E\ V0t) c_2Ebool_2E_7E$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2E$

Definition 14 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.(ap (ap$

Definition 15 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (ap (c_2Epred_set_2E$

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\forall V1s \in (2^{A_27a}).(((p (ap (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (\neg (V1s = (c_2Epred_set_2EEMPTY\ A_27a)))) \Rightarrow (p (ap V0P (ap (c_2Epred_set_2EREST\ A_27a)\ V1s)))) \Rightarrow (p (ap V0P\ V1s)))) \Rightarrow (\forall V2v \in (2^{A_27a}).(p (ap V0P\ V2v)))))) \quad (5)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}).((\forall V1s \in (2^{A_27a}).(((p (ap (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (\neg (V1s = (c_2Epred_set_2EEMPTY\ A_27a)))) \Rightarrow (p (ap V0P (ap (c_2Epred_set_2EREST\ A_27a)\ V1s)))) \Rightarrow (p (ap V0P\ V1s)))) \Rightarrow (\forall V2v \in (2^{A_27a}).(p (ap V0P\ V2v))))))$$