

# thm\_2Econtainer\_2ESET\_\_TO\_\_LIST\_\_THM (TMbFzG1AxjQAcQRbNSLhuRSVbBowwwXTmBf)

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Let  $c\_2Epred\_set\_2ECHOICE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_set\_2ECHOICE\ A\_27a \in (A\_27a)^{(2^{A\_27a})} \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1f \in 2.V1f)\ V0x)))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_2F)$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2IN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (2)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (3)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota) \Rightarrow \iota)$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow \forall A\_27b. nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}})$$
 (4)

**Definition 11** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota) \Rightarrow \iota)$

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota) \Rightarrow \iota)$

**Definition 13** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota) \Rightarrow \iota)$

**Definition 14** We define  $c\_2Epred\_set\_2EDELETE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1x \in A\_27a. (ap (ap c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota) \Rightarrow \iota)$

**Definition 15** We define  $c\_2Epred\_set\_2EREST$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (ap (c\_2Epred\_set\_2EDELETE : \iota \Rightarrow \iota) \Rightarrow \iota) \Rightarrow \iota)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0)$$
 (5)

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a})$$
 (6)

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a)$$
 (7)

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge p (ap P x)) \text{ of type } \iota \Rightarrow \iota)$

**Definition 17** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (ap (ap c\_2Ebool\_2E\_7E : \iota \Rightarrow \iota) \Rightarrow \iota) \Rightarrow \iota)))$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Ebool\_2EARB A\_27a \in A\_27a$$
 (8)

**Definition 18** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). (ap (c\_2Ebool\_2E\_21 : \iota \Rightarrow \iota) \Rightarrow \iota)$

**Definition 19** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 20** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 21** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 22** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_40 : \iota \Rightarrow \iota) \Rightarrow \iota)))$

**Definition 23** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 24** We define  $c\_2Erelation\_2ERESTRICT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1f$

**Definition 25** We define  $c\_2Erelation\_2ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

**Definition 26** We define  $c\_2Erelation\_2Eapprox$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 27** We define  $c\_2Erelation\_2Ethe\_fun$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 28** We define  $c\_2Erelation\_2EWFREC$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

**Definition 29** We define  $c\_2Elist\_2ESET\_TO\_LIST$  to be  $\lambda A\_27a : \iota.(ap (ap (c\_2Erelation\_2EWFREC (2^{A\_27a})$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p (ap \\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \Rightarrow ((ap (c\_2Elist\_2ESET\_TO\_LIST \\ A\_27a)\ V0s) = (ap (ap (ap (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a)) \\ (ap (ap (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ V0s)\ (c\_2Epred\_set\_2EEMPTY \\ A\_27a))) (c\_2Elist\_2ENIL\ A\_27a)) (ap (ap (c\_2Elist\_2ECONS\ A\_27a) \\ (ap (c\_2Epred\_set\_2ECHOICE\ A\_27a)\ V0s)) (ap (c\_2Elist\_2ESET\_TO\_LIST \\ A\_27a)\ (ap (c\_2Epred\_set\_2EREST\ A\_27a)\ V0s))))))) \end{aligned} \quad (9)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p (ap \\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \Rightarrow ((ap (c\_2Elist\_2ESET\_TO\_LIST \\ A\_27a)\ V0s) = (ap (ap (ap (c\_2Ebool\_2ECOND\ (ty\_2Elist\_2Elist\ A\_27a)) \\ (ap (ap (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ V0s)\ (c\_2Epred\_set\_2EEMPTY \\ A\_27a))) (c\_2Elist\_2ENIL\ A\_27a)) (ap (ap (c\_2Elist\_2ECONS\ A\_27a) \\ (ap (c\_2Epred\_set\_2ECHOICE\ A\_27a)\ V0s)) (ap (c\_2Elist\_2ESET\_TO\_LIST \\ A\_27a)\ (ap (c\_2Epred\_set\_2EREST\ A\_27a)\ V0s))))))) \end{aligned}$$