

# thm\_2Econtainer\_2EWF\_\_mlt\_\_list (TMaez7HzPGPuAiRjzHsCjiuskG7ZujZzdqL)

October 26, 2020

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 4** We define  $c\_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ecombin\_2EK\ ty\_2Enum\_2Enum))$

**Definition 5** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 6** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2y \in 2.V2y))\ P)$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Ecombin\_2EK\ m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2E\_2B) V0n)$

**Definition 10** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21) 2) (\lambda V0t \in 2. V0t)$ .

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2. V2t)))$

**Definition 14** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t3 \in 2. V2t3))))$

**Definition 16** We define  $c\_2Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0e V1b)$

**Definition 17** We define  $c\_2Ebag\_2EEL\_2BAG$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. (ap (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0e) V1b)$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21) V0t)$

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap V0P (ap (c\_2Emin\_2E\_3D\_3D\_3E) V0P)))$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap (c\_2Eprim\_rec\_2E\_3C) V0m V1n)$

**Definition 21** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap (c\_2Earithmetic\_2E\_3E) V0m V1n)$

**Definition 22** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2. V2t)))$

**Definition 23** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum. (ap (c\_2Earithmetic\_2E\_3E\_3D) V0m V1n)$

**Definition 24** We define  $c\_2Ebag\_2EBAG\_INN$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1n \in ty\_2Enum\_2Enum^{A\_27a}. (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0e V1n)$

**Definition 25** We define  $c\_2Ebag\_2EBAG\_IN$  to be  $\lambda A\_27a : \iota. \lambda V0e \in A\_27a. \lambda V1b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0e V1b)$

**Definition 26** We define  $c\_2Ebag\_2EBAG\_UNION$  to be  $\lambda A\_27a : \iota. \lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}). \lambda V1b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0b V1b)$

**Definition 27** We define  $c\_2Ebag\_2EFINITE\_2BAG$  to be  $\lambda A\_27a : \iota. \lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}). (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0b V1b)$

**Definition 28** We define  $c\_2Ebag\_2Emlt1$  to be  $\lambda A\_27a : \iota. \lambda V0r \in ((2^{A\_27a})^{A\_27a}). \lambda V1b1 \in (ty\_2Enum\_2Enum^{A\_27a}). (ap (c\_2Ebag\_2EBAG\_2EEL\_2BAG) V0r V1b1)$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (7)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (8)$$

Let  $c\_2Econtainer\_2ELIST\_TO\_BAG : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Econtainer\_2ELIST\_TO\_BAG\ A\_27a \in ((ty\_2Enum\_2Enum^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (9)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (10)$$

**Definition 29** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (11)$$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (12)$$

**Definition 30** We define  $c\_2Econtainer\_2Emlt\_list$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). (\lambda V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\lambda V2l2 \in (ty\_2Elist\_2Elist\ A\_27a). (ap\ V2l2\ (ap\ V1l1\ V0R))))$

**Definition 31** We define  $c\_2Erelation\_2Einv\_image$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27b})^{A\_27b}). \lambda V1l1 \in (ty\_2Elist\_2Elist\ A\_27b). (\lambda V2l2 \in (ty\_2Elist\_2Elist\ A\_27b). (ap\ V2l2\ (ap\ V1l1\ V0R))))$

**Definition 32** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}). (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0R)$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}). ((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V0b1)\ V1b2) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V1b2)\ V0b1)))))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\
& \text{nonempty } A\_27c \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).((ap (ap \\
& (c\_2Ebag\_2EBAG\_UNION A\_27a) V0b) (c\_2Ebag\_2EEMPTY\_BAG A\_27a)) = \\
& V0b)) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}).((ap (ap (c\_2Ebag\_2EBAG\_UNION \\
& A\_27b) (c\_2Ebag\_2EEMPTY\_BAG A\_27b)) V1b) = V1b)) \wedge (\forall V2b1 \in \\
& (ty\_2Enum\_2Enum^{A\_27c}).(\forall V3b2 \in (ty\_2Enum\_2Enum^{A\_27c}). \\
& (((ap (ap (c\_2Ebag\_2EBAG\_UNION A\_27c) V2b1) V3b2) = (c\_2Ebag\_2EEMPTY\_BAG \\
& A\_27c)) \Leftrightarrow ((V2b1 = (c\_2Ebag\_2EEMPTY\_BAG A\_27c)) \wedge (V3b2 = (c\_2Ebag\_2EEMPTY\_BAG \\
& A\_27c))))))))))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}). \\
& (\forall V1e \in A\_27a.((ap (ap (c\_2Ebag\_2EBAG\_INSERT A\_27a) V1e) \\
& V0b) = (ap (ap (c\_2Ebag\_2EBAG\_UNION A\_27a) (ap (c\_2Ebag\_2EEL\_BAG \\
& A\_27a) V1e)) V0b))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((p (ap (c\_2Erelation\_2EWF A\_27a) V0R)) \Rightarrow (p (ap (c\_2Erelation\_2EWF \\
& (ty\_2Enum\_2Enum^{A\_27a}) (ap (c\_2Ebag\_2Emlt1 A\_27a) V0R))))))
\end{aligned} \tag{16}$$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{19}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{20}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p \ V0P) \wedge (\forall V2x \in A\_27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \wedge (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (((ap \ (c\_2Econtainer\_2ELIST\_TO\_BAG \ A\_27a) \ (c\_2Elist\_2ENIL \ A\_27a)) = (c\_2Ebag\_2EMPTY\_BAG \ A\_27a)) \wedge \\ & (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist \ A\_27a). \\ & (ap \ (c\_2Econtainer\_2ELIST\_TO\_BAG \ A\_27a) \ (ap \ (ap \ (c\_2Elist\_2ECONS \ A\_27a) \ V0h) \ V1t)) = (ap \ (ap \ (c\_2Ebag\_2EBAG\_INSERT \ A\_27a) \ V0h) \ (ap \ (c\_2Econtainer\_2ELIST\_TO\_BAG \ A\_27a) \ V1t)))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0ls \in (ty\_2Elist\_2Elist \ A\_27a).(p \ (ap \ (c\_2Ebag\_2EFINITE\_BAG \ A\_27a) \ (ap \ (c\_2Econtainer\_2ELIST\_TO\_BAG \ A\_27a) \ V0ls)))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist \\ A.27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A.27a).((ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG \\ A.27a)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A.27a)\ V0l1)\ V1l2)) = (ap\ (ap\ ( \\ c\_2Ebag\_2EBAG\_UNION\ A.27a)\ (ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG \\ A.27a)\ V0l1))\ (ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG\ A.27a)\ V1l2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a.(\forall V1l \in \\ (ty\_2Elist\_2Elist\ A.27a).((p\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_IN\ A.27a) \\ V0h)\ (ap\ (c\_2Econtainer\_2ELIST\_TO\_BAG\ A.27a)\ V1l))) \Leftrightarrow (p\ (ap \\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\ A.27a)\ V1l)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\ (\forall V1P \in ((2^{A.27a})^{A.27a}).(((p\ (ap\ (c\_2Erelation\_2EWF\ A.27a) \\ V0R)) \wedge (\forall V2x \in A.27a.(\forall V3y \in A.27a.((p\ (ap\ (ap\ V1P\ V2x) \\ V3y)) \Rightarrow (p\ (ap\ (ap\ V0R\ V2x)\ V3y)))))) \Rightarrow (p\ (ap\ (c\_2Erelation\_2EWF\ A.27a) \\ V1P)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0R \in ((2^{A.27b})^{A.27b}).(\forall V1f \in (A.27b^{A.27a}).(\forall V2x \in \\ A.27a.(\forall V3y \in A.27a.((p\ (ap\ (ap\ (ap\ (c\_2Erelation\_2Einv\_image \\ A.27a\ A.27b)\ V0R)\ V1f)\ V2x)\ V3y)) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V1f\ V2x))\ (ap\ V1f \\ V3y)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0R \in ((2^{A.27b})^{A.27b}).(\forall V1f \in (A.27b^{A.27a}).(( \\ p\ (ap\ (c\_2Erelation\_2EWF\ A.27b)\ V0R)) \Rightarrow (p\ (ap\ (c\_2Erelation\_2EWF \\ A.27a)\ (ap\ (ap\ (c\_2Erelation\_2Einv\_image\ A.27a\ A.27b)\ V0R)\ V1f)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (50)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & ((p \ (ap \ (c\_2Erelation\_2EWF \ A\_27a) \ V0R)) \Rightarrow (p \ (ap \ (c\_2Erelation\_2EWF \\ & \quad (ty\_2Elist\_2Elist \ A\_27a)) \ (ap \ (c\_2Econtainer\_2Emlt\_list \ A\_27a) \\ & \quad \quad V0R))))) \end{aligned}$$